Radio Astronomy
Lecture 4:
Radio emission mechanisms and polarization

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9 April 2020

- Radio emission mechanisms
- Polarimetry

Last lecture.

Radio emission mechanisms (a quick observational discussion)

- What are the observational characteristics of different types of radio emission? How best to measure each type?
    - Thermal/blackbody
    - Gas lines and masers (bound-bound)
    - Bremsstrahlung (Free-free)
    - Synchrotron/cyclotron

Each mechanism has its own chapter in Essential Radio Astronomy, plus in emission mechanism books, so this will be fairly short.

Spectral index

- The most common way of describing radio spectra is as a power law, with a power law (spectral) index *α*:

$$I(\nu) = I_0 \left(\frac{\nu}{\nu_0}\right)^{\alpha}$$

- If the emission doesn't follow a power law, it can be described as spectral curvature, and the spectral index becomes frequency dependent: $\alpha(\nu) = \frac{\partial \log I(\nu)}{\partial \log \nu}$

- In brightness temperature units, since the conversion depends on frequency, the brightness temperature spectral index is different:

$$T_B = \frac{I(\nu)c^2}{2k\nu^2} \qquad T_B(\nu) = T_{B0} \left(\frac{\nu}{\nu_0}\right)^{\beta} \qquad \beta = \alpha - 2$$

Some define spectral index using -alpha for the exponent. This flips the sign of the spectral index (making synchrotron emission usually positive, instead of the normal negative spectral index). Always write what definition you're using,

Thermal (blackbody) radiation

- Produced from the internal energy of solid(-ish?) objects

$$I(\nu, t) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1} \approx \frac{2h\nu^2 kT}{c^2} \qquad \alpha = 2$$

- Not always so simple: modified blackbody laws, optically-thin blackbodies, etc, can change the spectrum (but I don't know the details).
- Because of the positive spectral index, generally only significant at very high frequencies.
- Unpolarized

In the low-frequency limit, the Planck equation can be simplified into the Rayleigh-Jeans equation, which predicts a spectral index of 2.

Thermal (blackbody) radiation

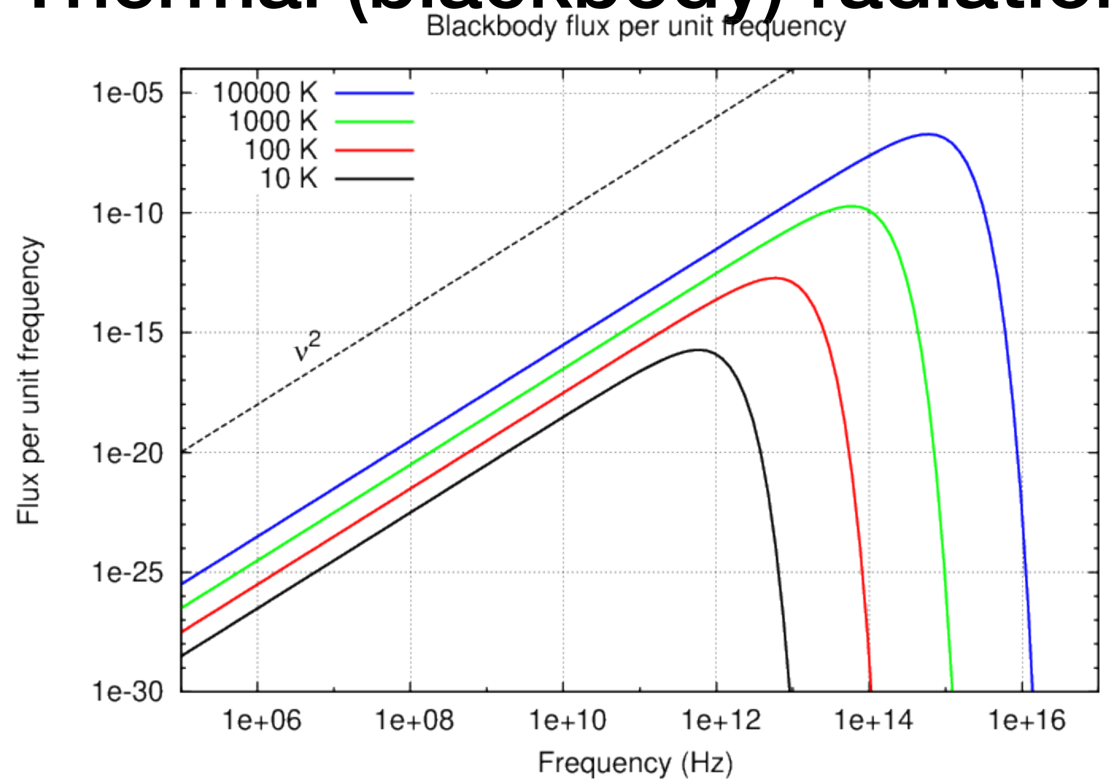


Image credit:Michael Richmond, RIT

Note how well the Rayleigh-Jeans approximation works, even for very low temperatures, up to over 100 GHz.

Gas lines/masers

- Atomic and molecular spectral lines, of various types (electronic state transitions, hyperfine splitting, ro-vibrational transitions, etc).
- Many lines have very narrow natural line-widths: mostly Doppler broadened allowing for measurements of velocity and velocity dispersion.
- Optically-thin lines provide column-densities; multiple lines from the same species can give temperature, density conditions
- Some lines experience Zeeman effect due to magnetic field, causing the lines to become polarized.

In most parts of the ISM, Zeeman splitting is too weak to fully split lines, so the result is broadened lines with complication polarization profiles (usually Stokes V)

Hydrogen 21cm hyperfine line

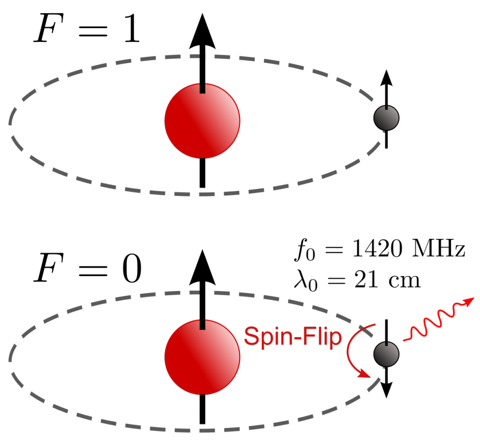


Image credit: Titec, Wikimedia Commons

- Neutral atomic hydrogen, in the ground state, has a hyperfine transition at 1 420 405 751.7667±0.0009 Hz
- Traces neutral hydrogen throughout the universe.
- Ultra narrow linewidth allows precise measurement of velocity distribution.
- Can also be seen in absorption in front of bright radio sources.

The mean lifetime of the spin-flip transition is 10 Myr. So the population in the 'excited' state is never depleted, and the number of photons is comparatively small relative to the number of hydrogen atoms.

CO lines

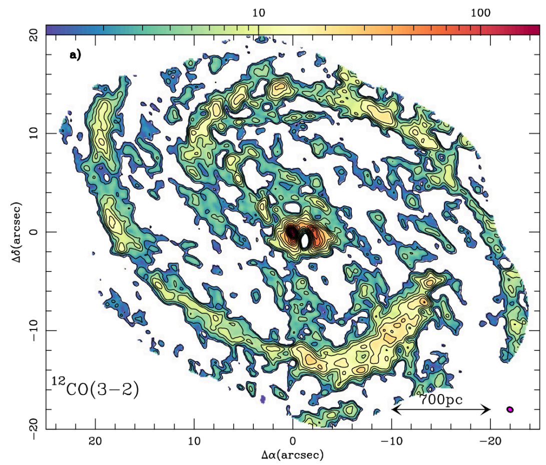


Image credit: García-Burillo et al. (2014)

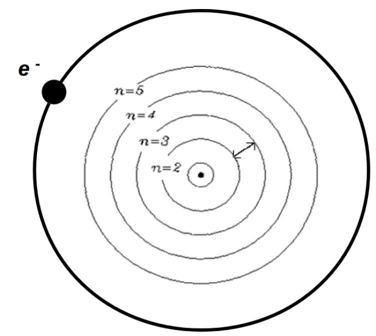
- Symmetric molecules don't produce rotational line transitions, so they have very few lines. The most common molecule, H_2, is symmetric and thus nearly invisible.
- CO molecules are also somewhat common, and have rotational transitions, at approximate multiples of 115 GHz
- Assuming some abundance relationship between CO and H_2 (X_{CO}), the H_2 mass, and thus total gas mass can be estimated.

CO J=1->0 is at 115 GHz, J=2->1 is at 230 GHz, etc.

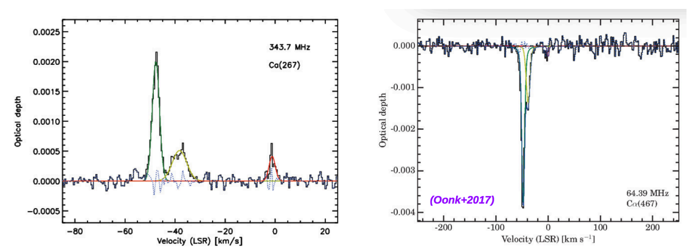
There's a whole slew of papers on X_{CO} and what values it has in different environments and how reliable it is and so on.

Also, isotopologues (molecules containing 1 or more atoms of less usual isotopes, like ^{13}C or ^{18}O) can provide additional information (often by being optically thin when the main lines become optically thick).

Radio recombination lines



11 MHz (n=843): r ~ 1 micron !



Cas A (WSRT P-band) Cas A (LOFAR LBA)

Image credit: Raymond Oonk

- Recombination (from plasma to gas) leaves the electron in an excited state, which cascades down to the ground state.
- Sometimes these will be at extremely large *n*, which produces weak lines in the radio
- Trace cold phase ISM, can measure temperature and density

Recombination lines are what produce strong H-alpha/Balmer series lines in HII regions.

I think these are cool because these are super extreme states of atoms, with electrons just barely bound.

Free-free/Bremsstrahlung

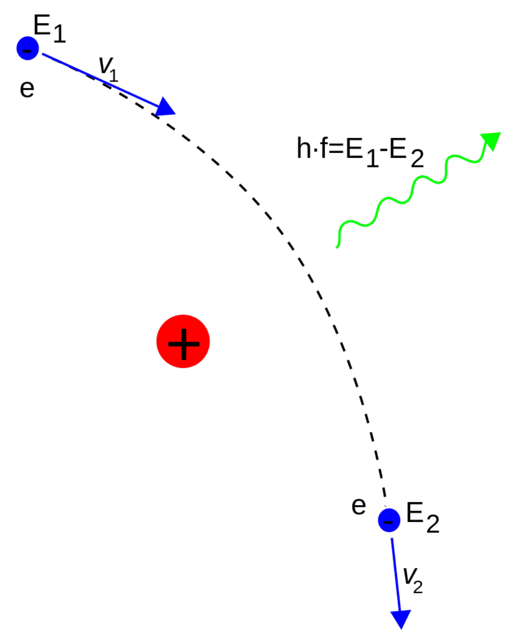
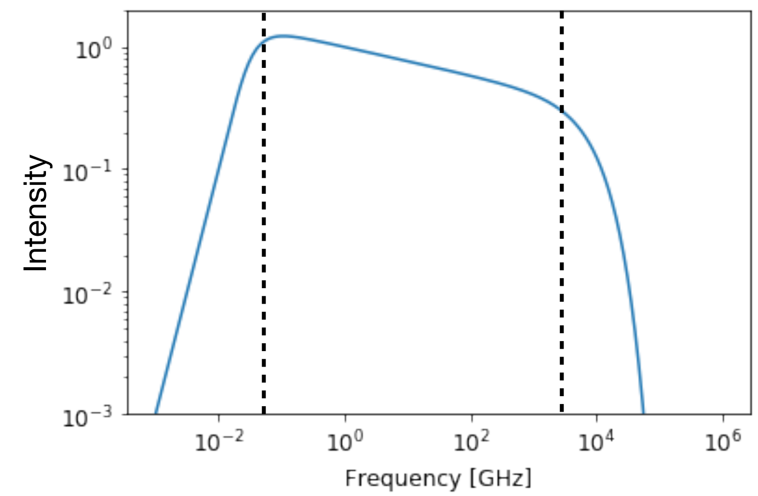


Image credit: Wikimedia Commons

Optically Thick $I \propto \nu^2$

Optically Thin $I \propto \nu^{-0.1}$

Thermal cutoff $I \propto e^{-h\nu/kT}$



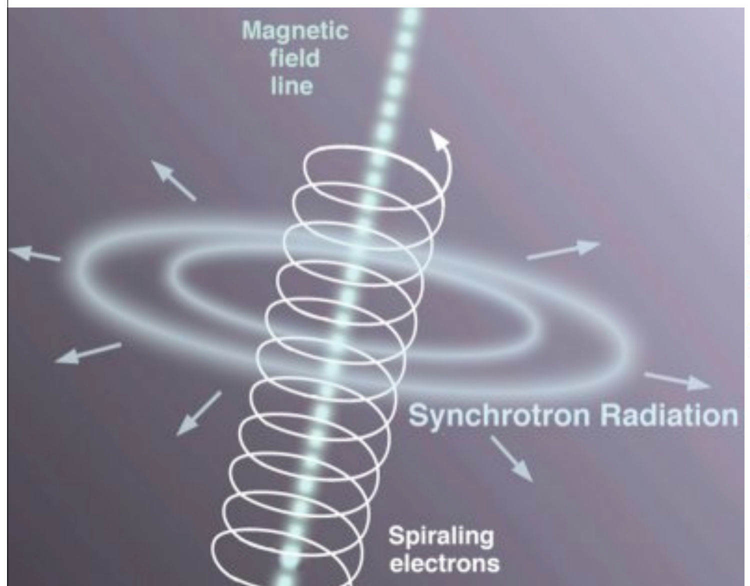
Free-free emission occurs in dense plasma environments (typically HII regions), where free electrons and ions can interact. Accelerating electrons produce broad-band radiation. Intensity depends on interaction rate, which goes as density squared.

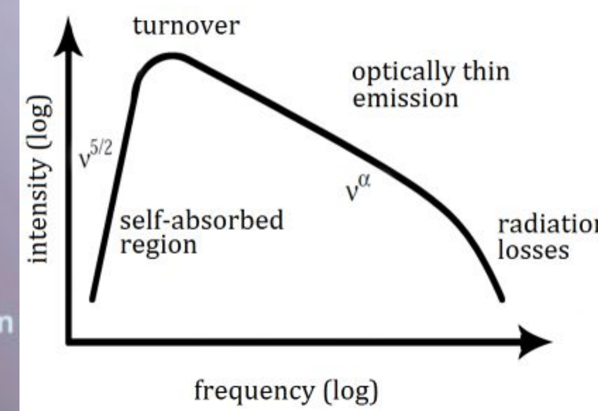
Free-free dominates mostly between about 30-200 GHz; below that synchrotron becomes bright and above thermal emission increases.

Fun fact: HII regions can have negative brightness in an interferometer

- Consider an HII region in the Galaxy at low frequencies where it is optically thick. Behind it is diffuse synchrotron emission from the Milky Way, which can have a higher brightness temperature at low frequencies.
- Because it's optically thick, it absorbs the background radiation.
- An interferometer removes the 'mean'/smooth emission from the sky, which is the diffuse synchrotron emission.
- Since the HII region has lower brightness temperature, when the mean is removed it has a negative value in the interferometer image.

Synchrotron/cyclotron





$$-3 \lessapprox \alpha \lessapprox 0$$

The spectral index of synchrotron radiation (in the optically thin regime) depends on the distribution of energy of the cosmic ray electrons. There's a relationship between the power law index of electron energy and synchrotron radiation, so spectrum says something about electron population.

Synchrotron

- Intensity depends on cosmic ray electron density, and magnetic field strength. Spectral index depends on electron energy distribution.

- Making some assumptions about CR density (e.g., equipartition of energy between CRs and magnetic field), it’s possible to estimate magnetic field strength of synchrotron source.

- Synchrotron emission is intrinsically beamed perpendicular to the magnetic field, so it’s sensitive to only that part of the magnetic field.

- Synchrotron is polarized perpendicular to magnetic field; strength of polarization depends on spectral index and degree of order in the magnetic field.

Degree of order dependence is because synchrotron emission is produced over some volume. If the magnetic field orientation changes inside that volume, then the polarization produced in different parts of the volume will be different. If they are averaged together, this produces depolarization.

Polarimetry

- Since antennas are intrinsically sensitive to only one polarization state, understanding polarization is necessary to fully understand the behaviour of the telescope
- Measuring polarization adds additional information on the physics of the source/line-of-sight.
- Polarization generally requires some kind of symmetry breaking; often this is given by a magnetic field.

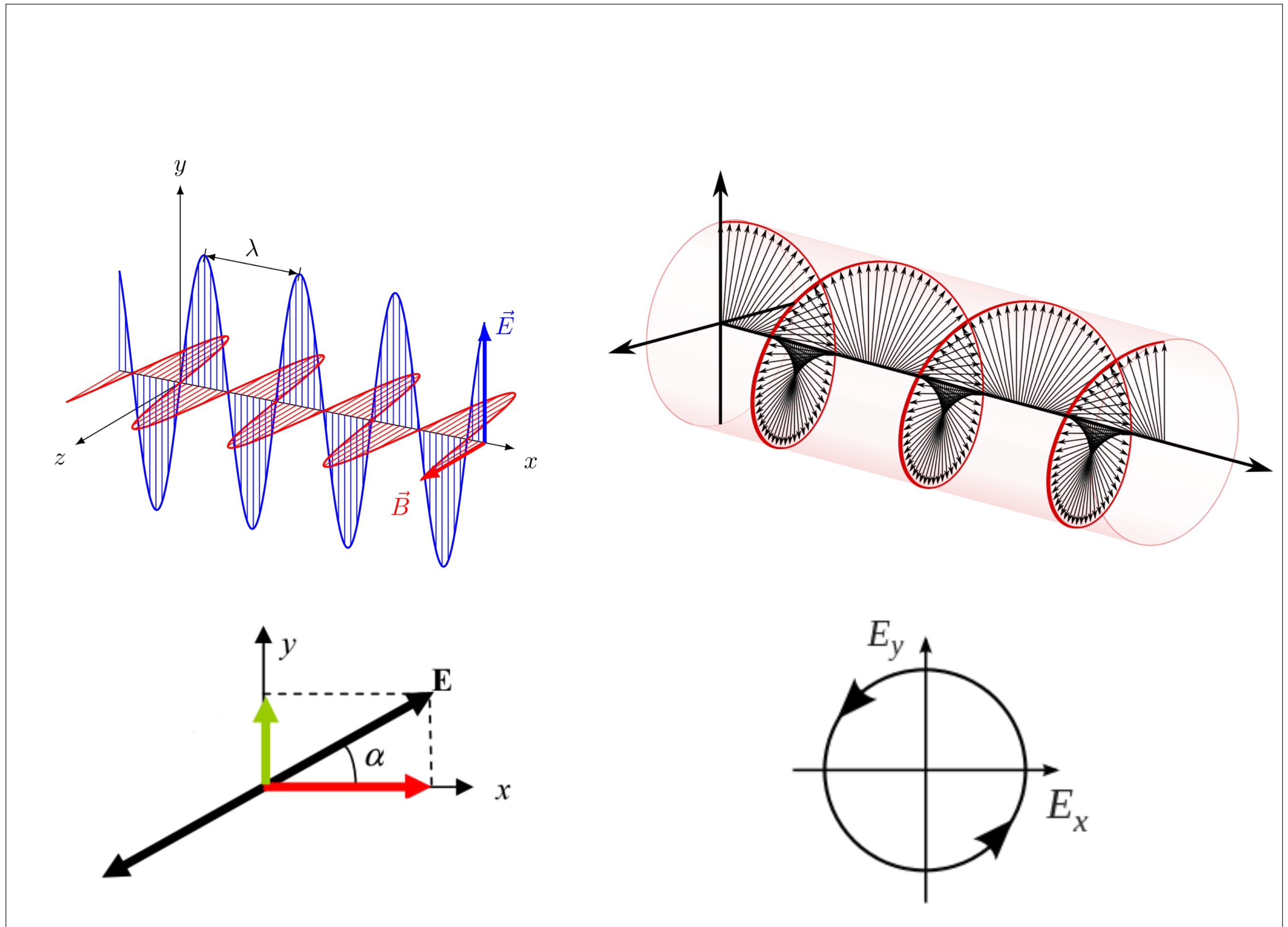
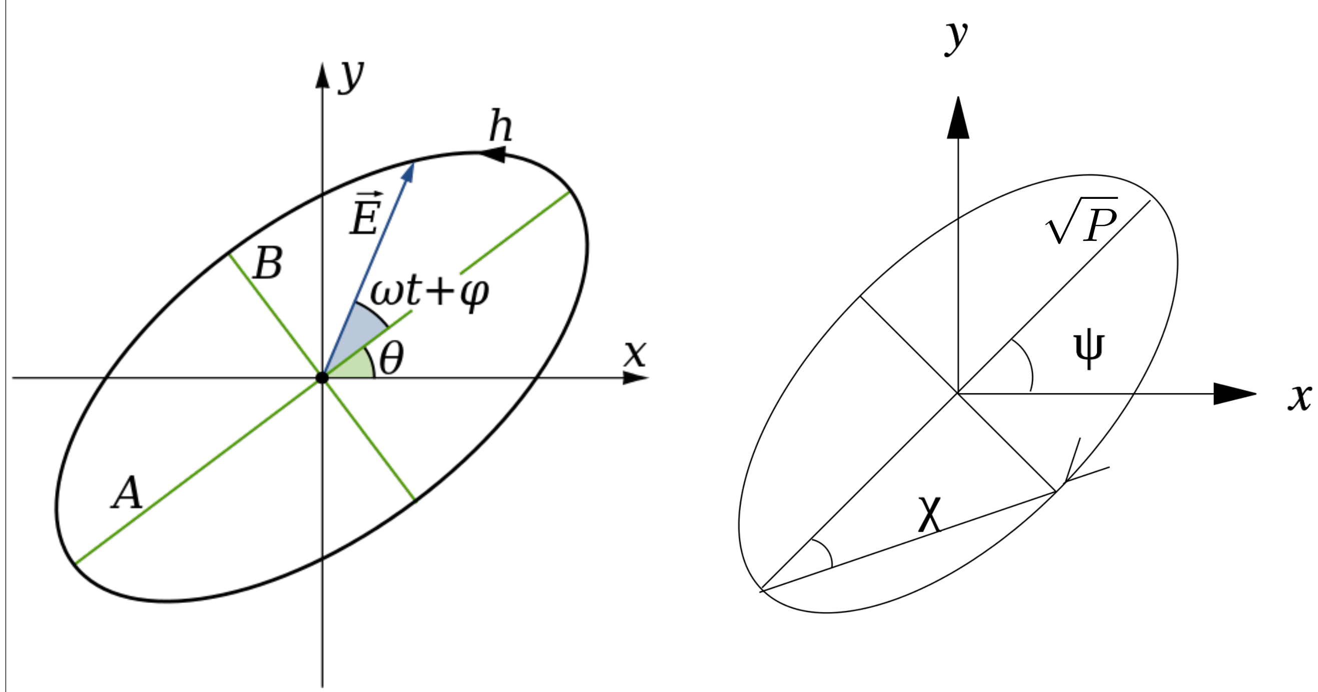


Image credit: Wikimedia Commons

There are two equally valid sets of basis states for polarization: *linear* polarization (with x and y as the two orthogonal modes), and *circular* polarization (with left-handed and right-handed as the orthogonal modes). Both bases can be used to describe all possible (pure*) polarization states, although the linear basis is easier to visualize for most people.

*:A ‘pure’ polarization state is one that is constant over time, and corresponds to 100% polarization. In contrast with partially polarized/unpolarized states, where the polarization is effectively random over time.

Polarization ellipse

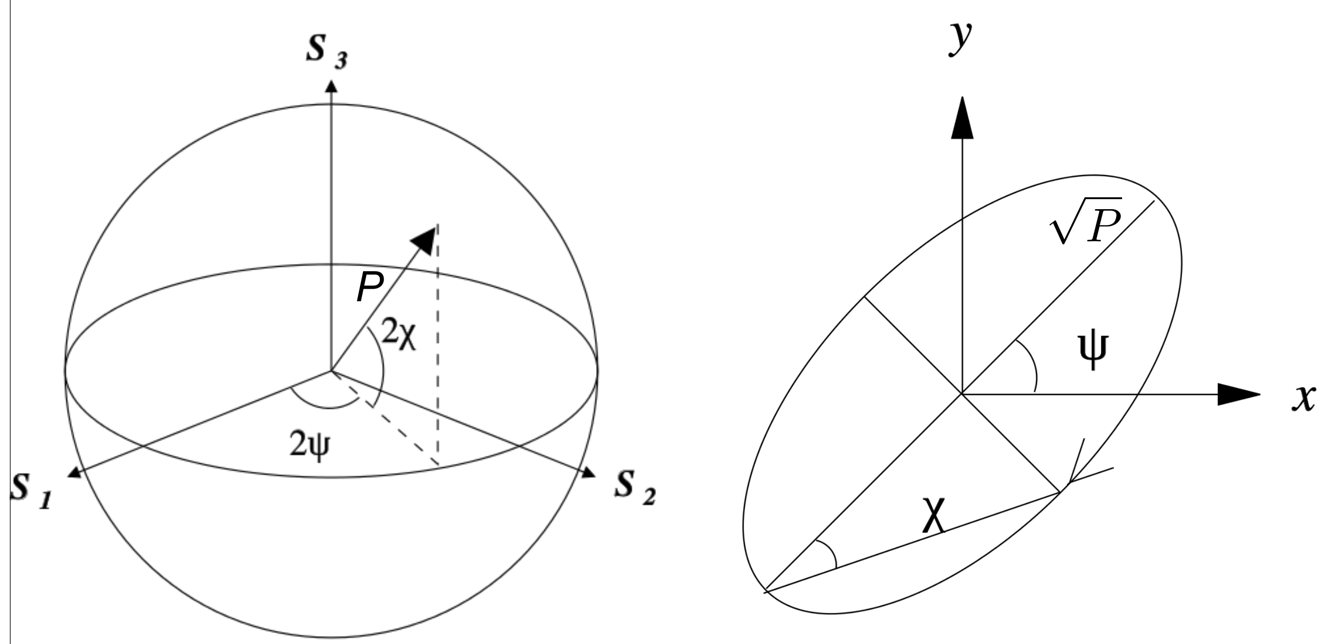


The general form of a polarized wave is the *polarization ellipse*. The electric field vector traces out one loop around the ellipse each wavelength or period. There are many ways of defining an ellipse, so there are several different sets of parameters that can be used.

On the left: (semi-)major axis, (semi-)minor axis, and polarization angle (A, B ,θ). Semi-minor axis has to be a signed quantity in order to carry information on left vs right circular polarization.

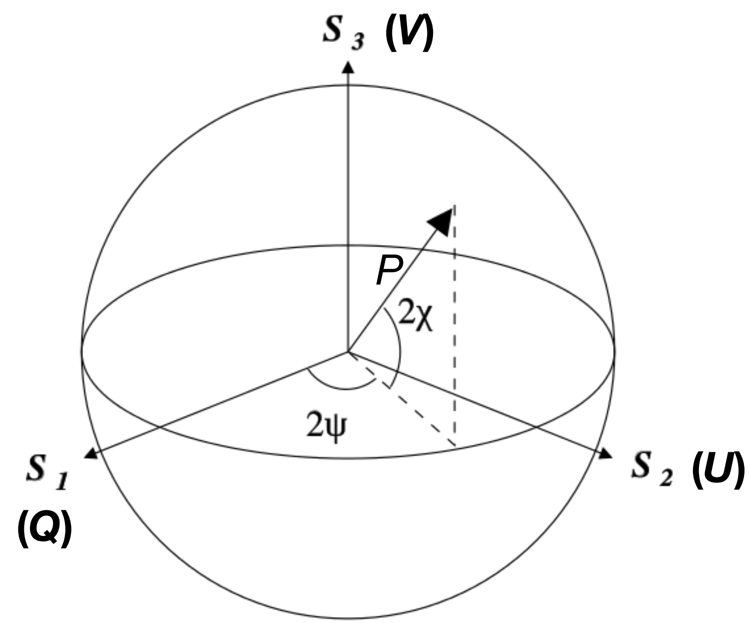
On the right: polarized intensity, ellipticity angle, and polarization angle (*P*, χ, ψ). Sqrt(*P*) shows up because the polarization ellipse is in electric field units, but *P* is in power/intensity units. The ellipticity angle can also be used to encode left vs right circular ellipses (as positive or negative angle)

Polarization ellipse



Note that polarization angle is only defined in a 180 degree range: an ellipse with ψ = ψ + 180° is identical (but with 180° phase shift).
Also, ellipticity angle is defined by the ratio of semi-minor to semi-major axis (with sign giving left- vs right-handed circular), so can only take values between -45° and 45°.
Together, P, 2χ, and 2Ψ form a set of spherical coordinates, which define the Poincaré sphere, which is the parameter space of all possible polarization states.
States on the equator represent pure linear polarization states (ellipticity = 0). The poles represent purely circular polarization.

Poincaré sphere



The spherical representation means there's a corresponding cartesian coordinate system: the Stokes parameters (*Q, U, V*).
+*Q* corresponds to horizontal polarization, -*Q* to vertical polarization (remember, 2Ψ).
+*U* corresponds to 45° polarization angle, -*U* to -45°/135°.
+*V* correspond to right circular polarization, -*V* to left circular (IEEE standard?). The exact definition of what is left vs right depends on convention: physicists and engineers have different conventions, and radio astronomy usually follows engineering (IEEE) convention.

Stokes parameters

- Stokes parameters can be instantaneously for a wave, but we usually measure the time-average. Since the parameters can vary with time and potentially cancel out, this leads to a 4th parameter: intensity (Stokes I).

- Unpolarized emission can have well-defined intensity, but no net polarization: $I \neq 0$, Q=U=V=0.

- Correlation products (visibilities, single-dish auto/cross correlations) can be defined in terms of Stokes parameters, and vice versa.

- $I = XX + YY$
- $Q = XX - YY$
- $U = XY + YX$
- $V = i\,(XY - YX)$

- $XX = (I + Q) / 2$
- $YY = (I - Q) / 2$
- $XY = (U - i\,V) / 2$
- $YX = (U + i\,V) / 2$

- $I = RR + LL$
- $Q = RL + LR$
- $U = i\,(RL - LR)$
- $V = RR - LL$

- $RR = (I + V) / 2$
- $LL = (I - V) / 2$
- $RL = (Q - iU) / 2$
- $LR = (Q + iU) / 2$

These are the conversions, for reference. It's interesting to note that they're all simple linear combinations, so as long as each of the correlations has independent noise then the noise in the Stokes parameters is well behaved.
It should be noted that if X and Y are defined in the frame of the antenna, then the Stokes parameters will be as well. This necessitates a conversion into the sky frame of reference (which only affects Q and U) in order to get correct polarization angles and to remove parallactic angle effects. This is usually done automatically in software.

- I = XX + YY
- Q = XX - YY

- I = RR + LL
- V = RR - LL

- One practical aspect: traditionally, Stokes I models are used for primary calibration (thus using the XX and YY/RR and LL data). In some cases, this can push calibration problems/bad data into Stokes Q (linear feeds) or V (circular feeds), causing problems. Thus Q or V were used to check for signs of bad calibration.

Sometimes the solver finds calibration solutions that make Stokes I look OK, but leaves problems that then show up in Q or V.

Polarization calibration

- Can be up to 4 steps required for polarization calibration*:
  - X-Y/R-L phase difference
  - On-axis (direction-independent) polarization leakage
  - Off-axis (direction-dependent) polarization leakage
  - Ionospheric Faraday rotation correction

*: there may be some that I am no aware of.

Polarization calibration

- X-Y/R-L phase difference
  - Stokes I calibration depends only on ‘same-hand’ correlations, so X and Y/R and L calibrated independently.
  - Phase difference for ‘cross-hand’ correlations affects other Stokes: for linear feeds, *U* and *V*; for circular, *Q* and *U*.
  - Solving for phase difference requires calibrator with known Stokes parameters.

Polarization calibration

- On-axis polarization leakage (D-terms):
  - Represent coupling between different polarizations in the antennas and electronics.
  - Cause signal from one Stokes parameter to show up in other Stokes.
  - Requires calibrator of known Stokes parameters (could be unpolarized)

Traditionally, leakage calibration was often done with known unpolarized sources. I’m not sure if that was due to software limitations (inputting a source model wasn’t supported?) or because ionospheric calibration wasn’t available (which changes the polarization measured).

Polarization calibration

- Off-axis polarization leakage (polarized beam):
  - Direct-dependent polarization effects that come from the primary beam (dish imperfections, antenna effects, etc).
  - Generally can't be directly calibrated for (too many free parameters!). Polarized beam can be measured from holography data, or estimated by fitting smooth function to the data.

Polarization calibration

- Ionospheric Faraday rotation
  - Changes the polarization from the astrophysical values, in a time- and position-dependent way
  - Predominantly affects low-frequency observations (<~ 1 GHz)
  - Cannot be done* using just a radio observation; requires external measurements (of ionospheric electron content)

*: For circular feeds, computing the X-Y phase difference as a function of frequency has the net effect of calibrating out the ionosphere

Polarization measurements

- Stokes parameters can be computed in visibility space as combination of correlations, then pushed through imaging process to get Stokes images (this was already happening for Stokes I, we just skipped that detail)

- A full set of Stokes images/cubes (*IQUV*) gives full polarization information as a function of position (and frequency).

- All the Stokes parameters are subject to the same imaging aspects: Clean, artifacts, noise, etc.

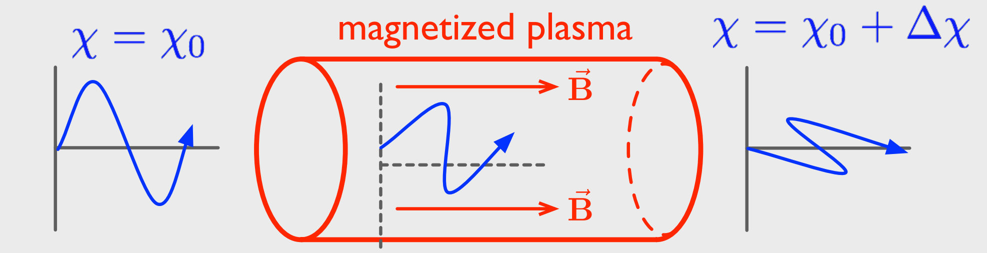
Polarization measurements

- Some related parameters/calculated quantities:
  - Linear polarized intensity: $P_{\rm lin} = \sqrt{Q^2 + U^2}$
  - Total polarized intensity: $P_{\rm tot} = \sqrt{Q^2 + U^2 + V^2}$
  - (Linear) Polarized fraction: $p = P_{\rm lin}/I$
  - Fractional Stokes parameters: $q = Q/I \;\; u = U/I \;\; v = V/I$
  - Circular polarized fraction: $p_{\rm circ} = |V|/I = |v|$
- Polarization angles are defined, in the sky frame, as East from North: 0° is towards the NCP, 90° is east (positive RA). Note that cosmologists sometimes use opposite direction (west from north), which has the effect of reversing Stokes *U* (U -> -U).

Worth noting, but beyond the scope of this course: since many of these are non-linear combinations of Stokes parameters (particularly polarized intensity), the error properties can be a little messy/complicated. For example, since $P_{\rm lin}$ is positive definite, it has a noise-bias (it tends to be higher than the true value, in low to intermediate signal-to-noise measurements).

Faraday rotation

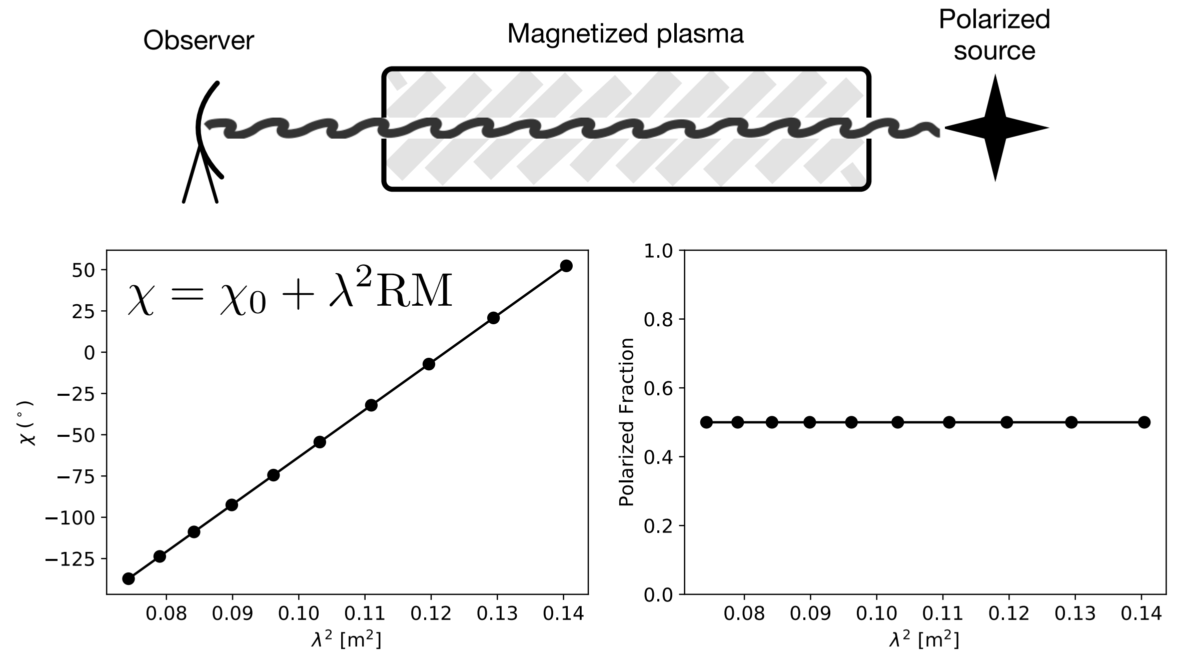
- Magnetized plasma is birefringent: different polarizations have different indices of refraction, and thus different phase velocities. Specifically, left- and right-circular polarized waves have different phase velocities.
- As waves propagate through the medium, the relative phase changes, changing the polarization angle (if linearly polarized).



$$\Delta\chi = \lambda^2 \left(0.81 \int n_e \vec{B} \cdot d\vec{l} \right) = \lambda^2 \phi$$

The derivation of birefringence and Faraday rotation in plasma appears in most plasma physics textbooks, so I won't go through it. The value of the integral is traditionally called the rotation measure, or Faraday depth (distinction will be made later).

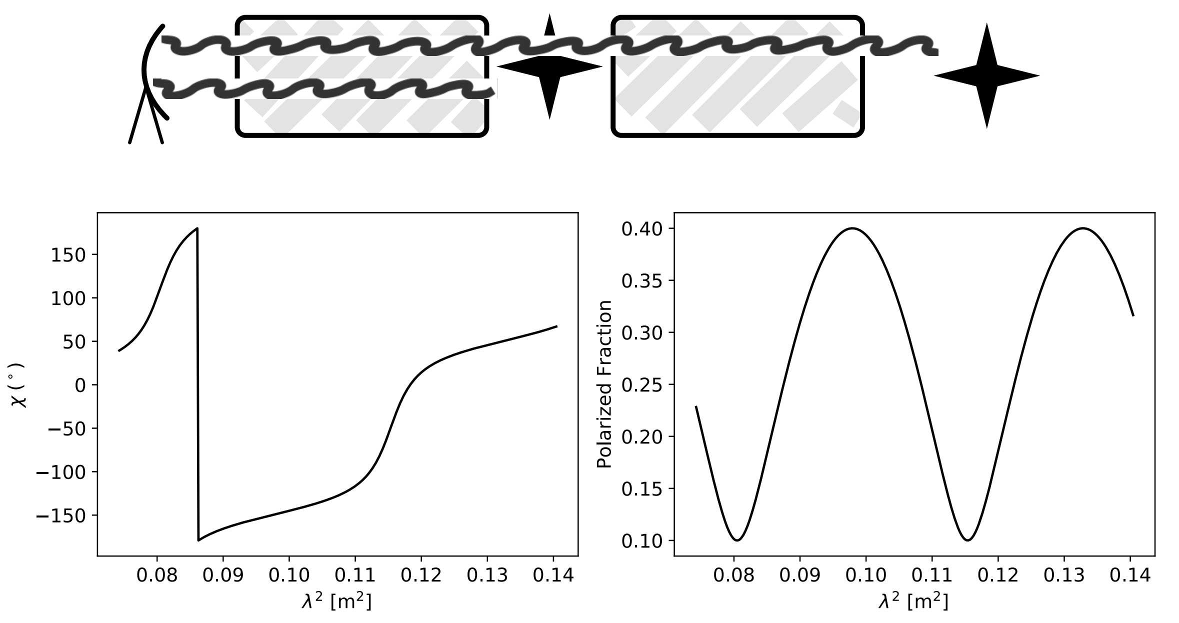
Faraday rotation



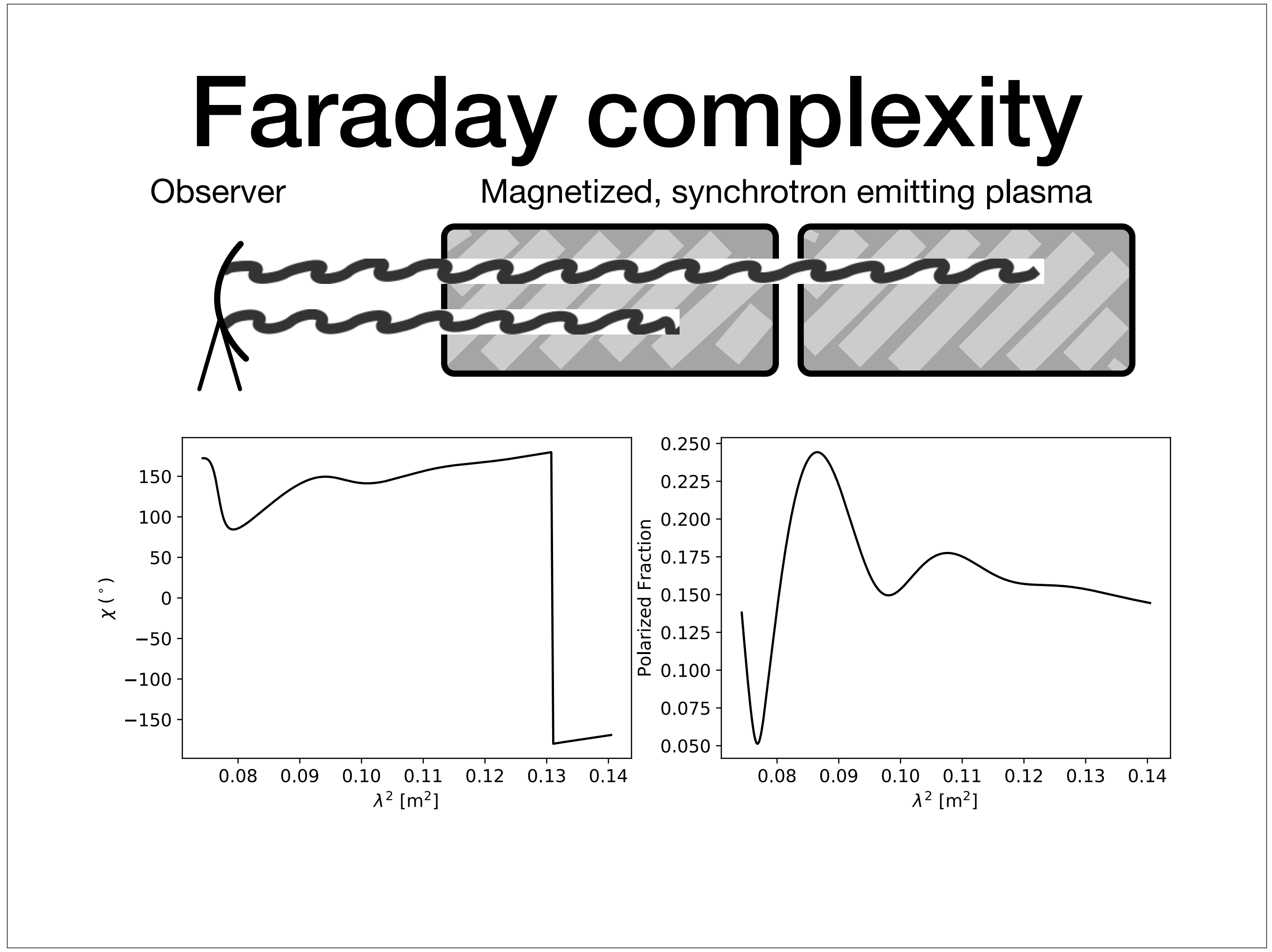
Classical method: Measure χ vs λ^2, fit the slope ϕ.

The traditional method of determining rotation measures is to observe polarization at different frequencies, and fit a straight line to polarization angle vs wavelength squared.

Faraday complexity



The traditional method breaks down when you have a line of sight with more than one source of polarized emission. The linearity of polarization angle vs wavelength squared is broken.



This breaking of the linearity becomes even worse if you have emission that is distributed over distance (such as diffuse polarized synchrotron emission).

Rotation measure synthesis

- Define the complex polarization as $\tilde{P} = Q + iU = Pe^{i2\chi}$
- We can then write Faraday rotation as $\tilde{P}(\lambda^2) = Pe^{i2\chi_0}e^{2i\lambda^2\phi}$
- What we observe is the superposition of all polarized emission at all distances:

$$\tilde{P}(\lambda^2) = \int_0^d \tilde{P}(r,\lambda^2)dr = \int_0^d P(r)e^{2i\chi_0(r)}e^{2i\lambda^2\phi(r)}dr$$

- Since there's a mapping from distance to Faraday depth:

$$\phi(d) = \int_0^d n_e B_{\|} dr \qquad \tilde{P}(\lambda^2) = \int_{-\infty}^{\infty} P(\phi)e^{2i\chi_0(\phi)}e^{2i\lambda^2\phi}d\phi$$

Doesn't this last equation look an awful lot like a Fourier transform?

For more on the mapping from distance to Faraday depth, see Van Eck (2018)

Rotation measure synthesis

$$\tilde{P}(\lambda^2) = \int_{-\infty}^{\infty} P(\phi) e^{2i\chi_0(\phi)} e^{2i\lambda^2\phi} d\phi$$

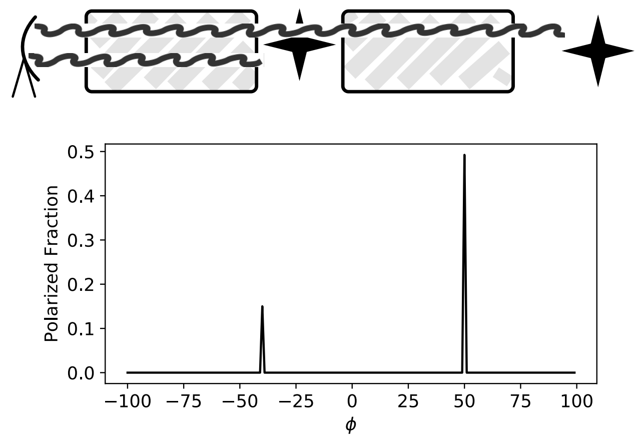
$$\tilde{P}(\phi) = \int_{-\infty}^{\infty} \tilde{P}(\lambda^2) e^{-2i\lambda^2\phi} d\lambda^2$$

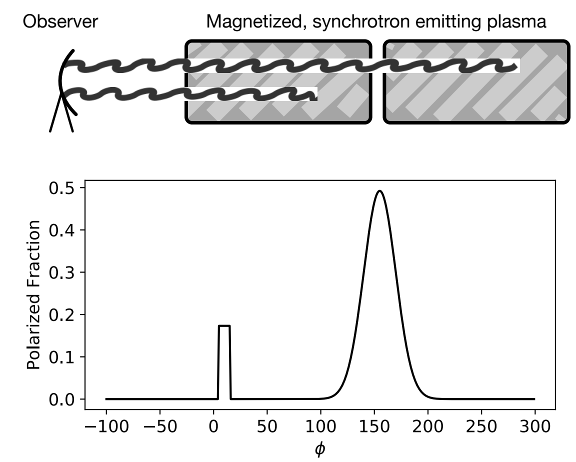
- Fourier relationship between polarization as function of wavelength squared and as function of Faraday depth.
- Incomplete sampling in wavelength squared domain leads to point spread function in Faraday depth domain (rotation measure spread function).

The properties of the RM spread function, and of RM synthesis in general, could fill an entire mini-course of their own.

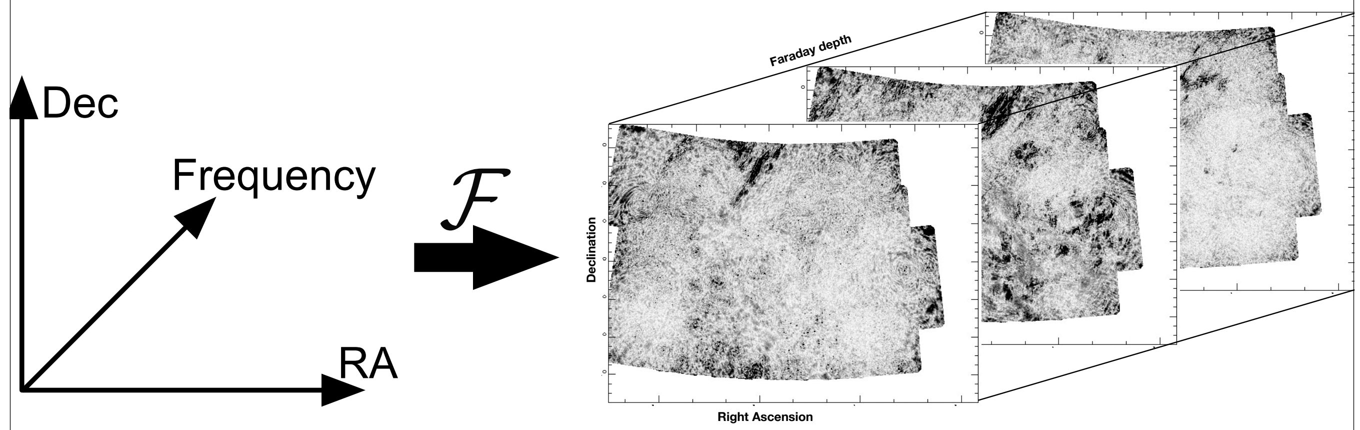
RM Synthesis

- The result is a ‘Faraday depth spectrum’, a measure of the polarization as a function of different possible Faraday depths. Thus, different emission regions can be identified and characterized (in principle).





Faraday tomography



Faraday tomography is the application of RM synthesis across all pixels in an image, producing 3D ‘Faraday depth cubes’ as outputs.

End of lectures. Thank you for reading these lecture notes. Please feel free to contact me with questions and comments.