Radio Astronomy Lecture 1: Basics

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12 Mar 2020

- Logistics
- Radio spectrum
- Down the signal chain of a radio telescope:
    - Antennas
    - Dishes and phased arrays
    - Signal processing: heterodyning, channelization, digitization
    - Radiometer equation

Logistics

- Syllabus link
- 4x lectures
- Designed to complement standard reference material (mostly 'Essential Radio Astronomy' by Condon and Ransom)

Logistics

- 1x Homework problem set
- I'm trying to make it fun/interesting, even for non-crediting students. Questions will be posted as they are developed. All questions will be posted before last lecture.
- Deadline will be 1 week after last lecture.
- Do it however you want: groups or individual, write it up however you want (any valid form of scientific communication), will be judged on correctness and clarity of thought.

Logistics

- 1x Tutorial + data processing exercise
- Tutorial will be held during one of the lecture timeslots, probably after 3rd lecture. Will cover introduction to data processing in CASA. Laptop with CASA installed recommended (otherwise, team up).
- Data processing exercise will involve finding an interesting data set on the VLA archive and trying to calibrate and image it and make a scientific measurement. Crediting students should submit a lab book/report detailing what they did and why.
- Deadline will be 1 week after last lecture.

Logistics

- Drop date 3 weeks from now.
  I need to know before that point who is crediting.

- Grade deadline: 2 weeks after last lecture (1 week after homework deadline)

- All course materials posted online at https://cameron-van-eck.github.io/teachradio2020.html

Radio Astronomy

Image credit: MPIfR-Bonn

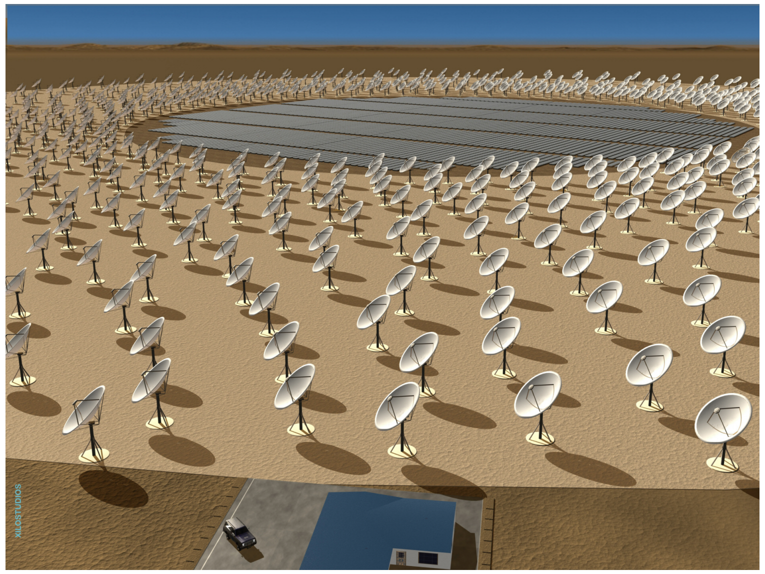
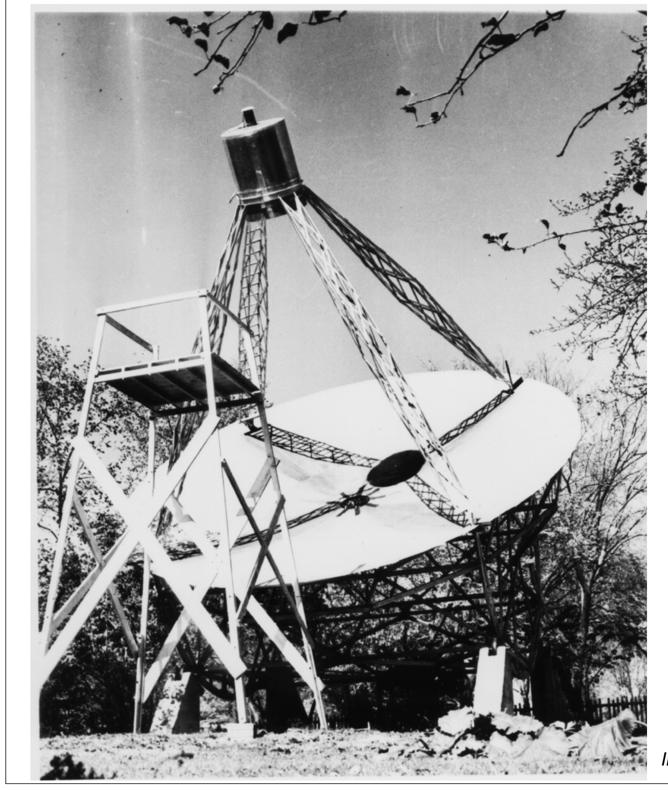


Image credit: XILOSTUDIOS/SKA Consortium

History



- Karl Jansky: first detection, 1932
- Grote Reber: first purpose-built radio telescope, 1937
- Hendrik van de Hulst: predicted 21cm HI line, 1944
- etc., etc.

Image credit: Wikimedia Commons

All radio courses start with the history of radio astronomy. I don’t care that much, so let’s skip it. The picture is Grobe Reber’s radio telescope.

Atmospheric Windows

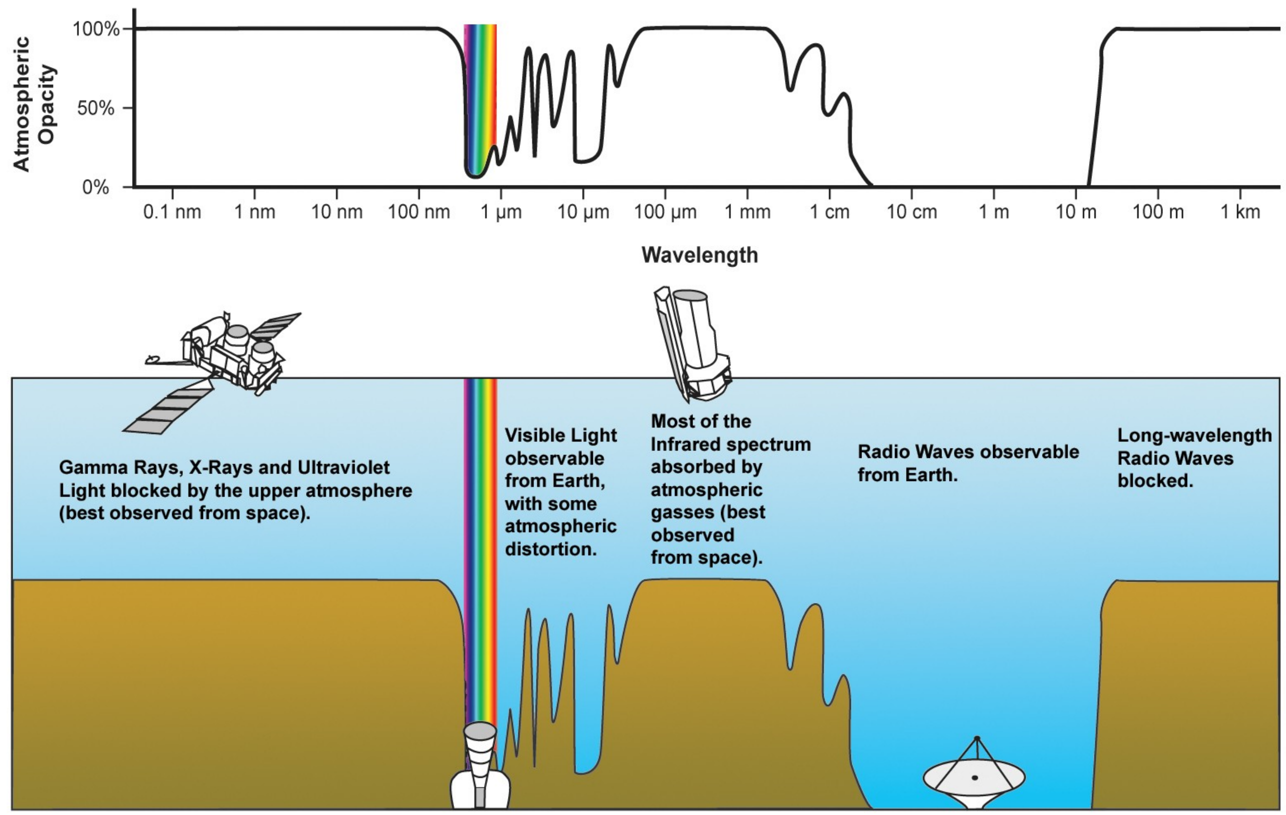


Image credit: NASA/Wikimedia Commons

Why is radio special? There are two atmospheric windows: the optical, and the radio. The radio window is huge, 3-4 orders of magnitude. Not needing a satellite means you can buy a lot of telescope for the same cost.

Atmospheric Windows

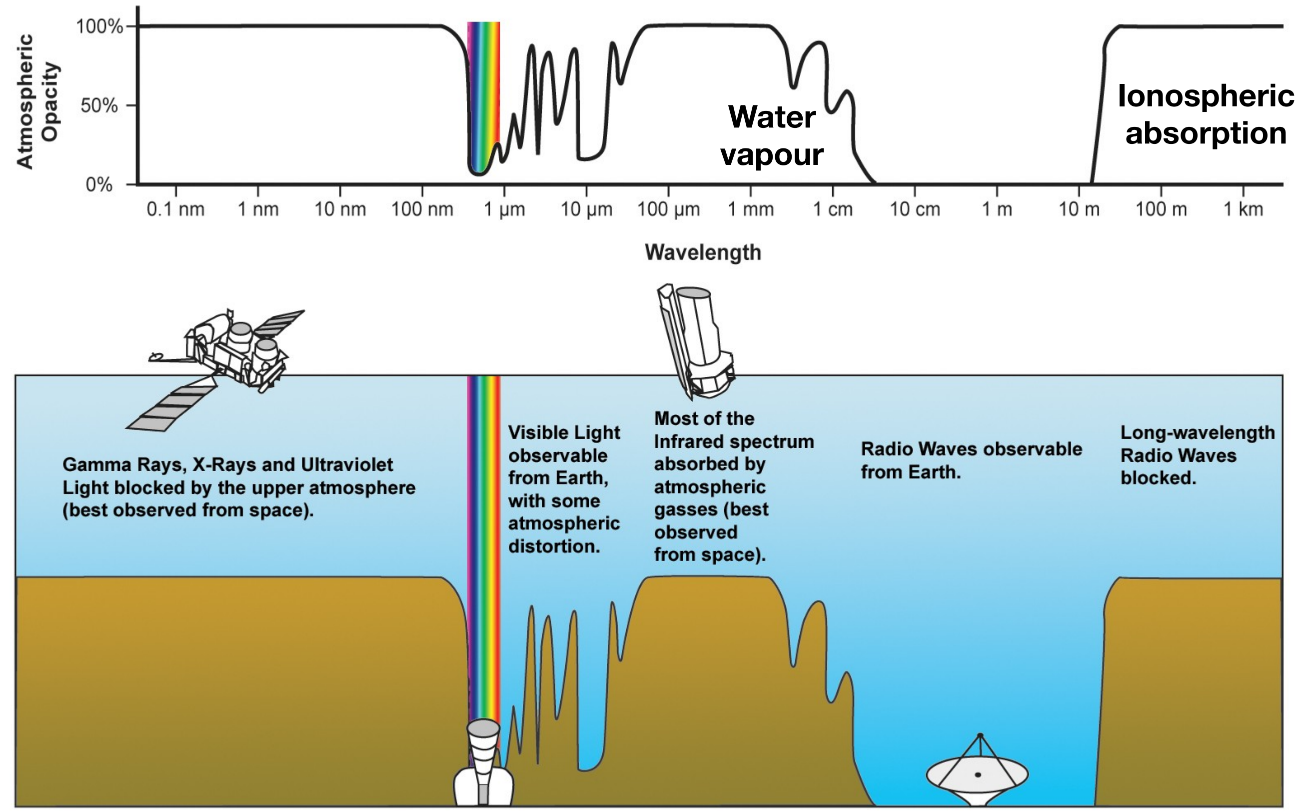


Image credit: NASA/Wikimedia Commons

The radio window is limited by two physical processes. At the short end, by absorption by water vapour (molecular lines). This can be overcome by high-altitude sites, so we can actually go much further than the plot says: into the sub-mm regime.
At the low end, plasma absorption by the ionosphere. It depends on electron density in the ionosphere, so some variation by day or night, but all longer wavelengths are absorbed.

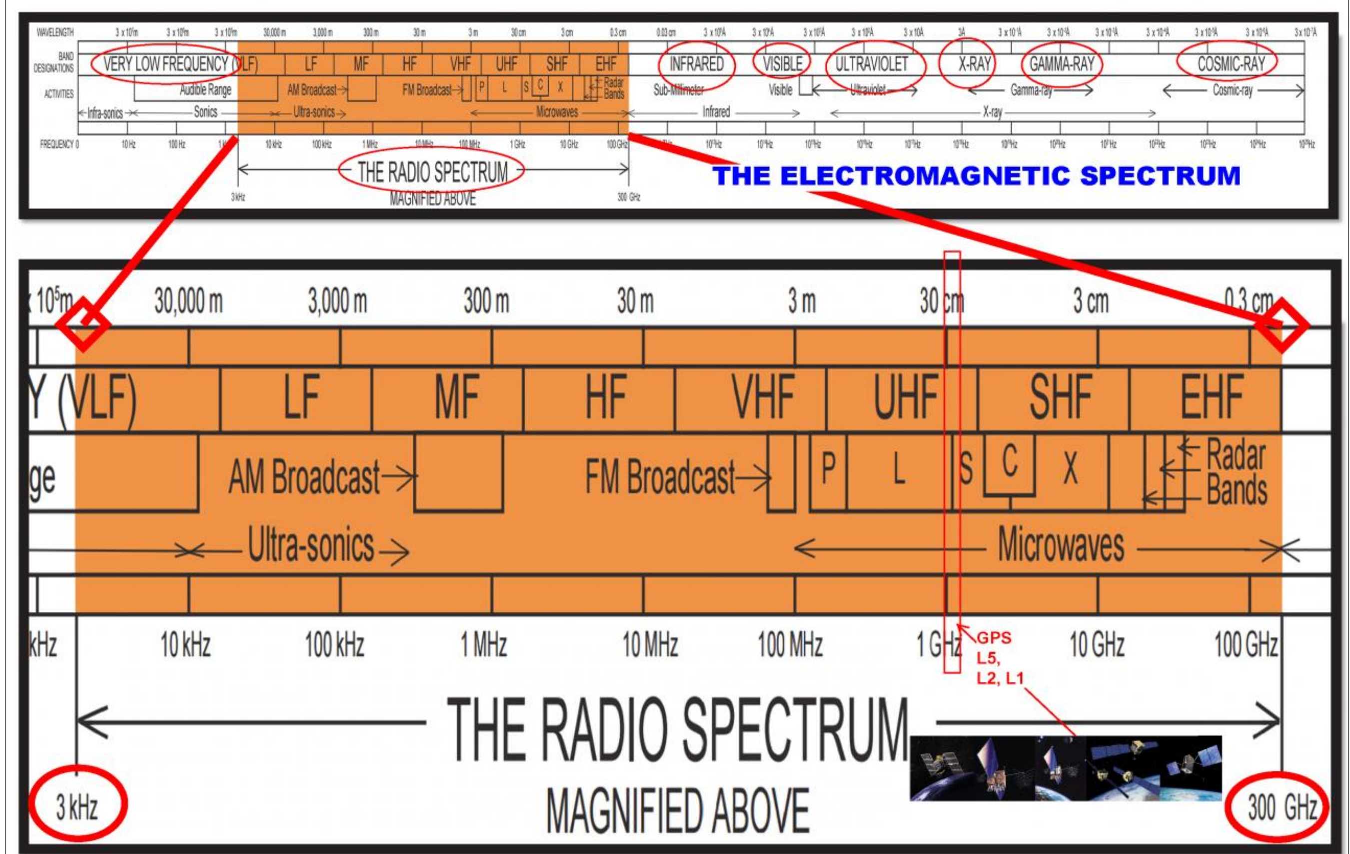


Image credit: US Dept. of Transportation

Most radio astronomy is talked about in terms of frequency, although wavelength is sometimes used interchangeably. Good to memorize a reference point, e.g. 1420 MHz = 21 cm, 300 MHz = 1 m, or 10 GHz = 3 cm.

Some people like to break spectrum into radio ‘bands’: P, L ,S, C, X, K, Ku, etc. Not worth memorizing until you need to.

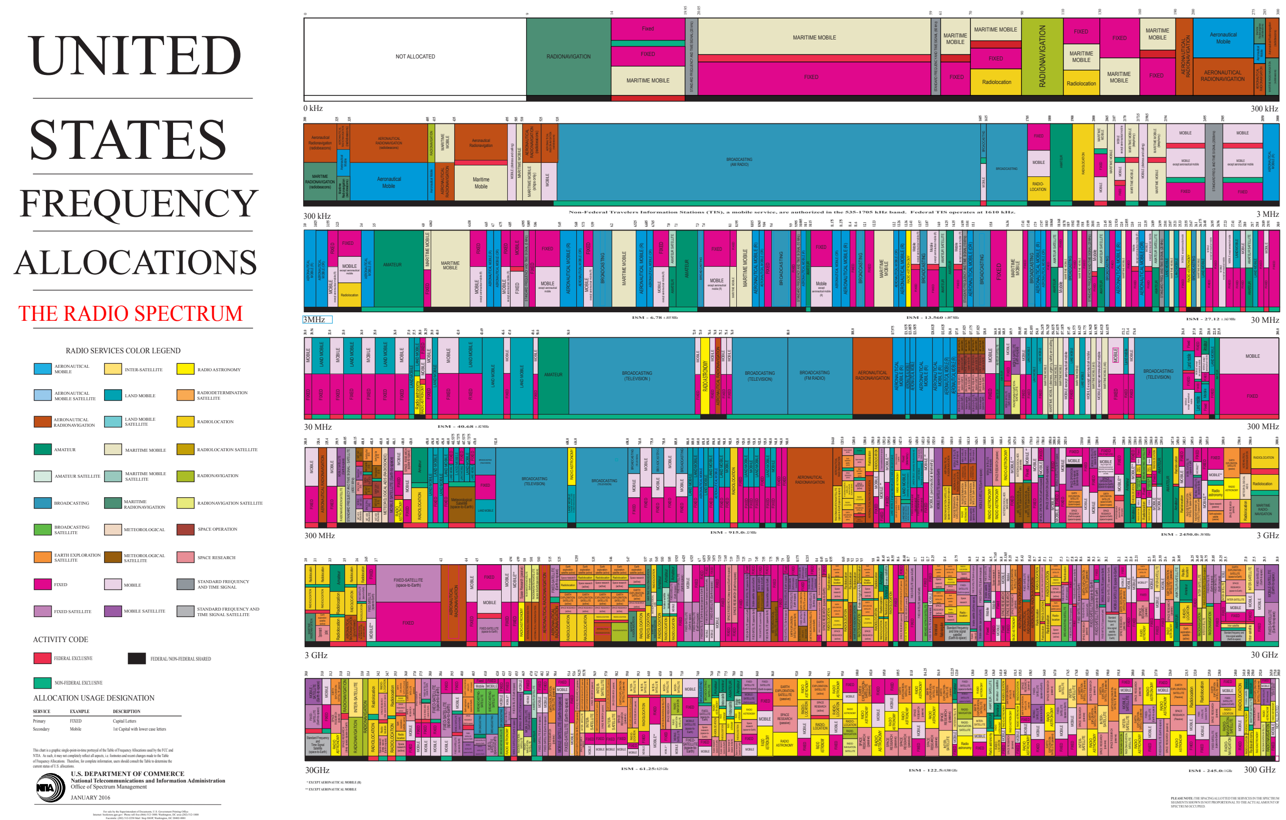


Image credit: US Dept. of Commerce, Office of Spectrum Management

But we don't get all that spectrum to ourselves. Governments determine how different parts of the spectrum are used. Parts get sold off periodically through auctions.

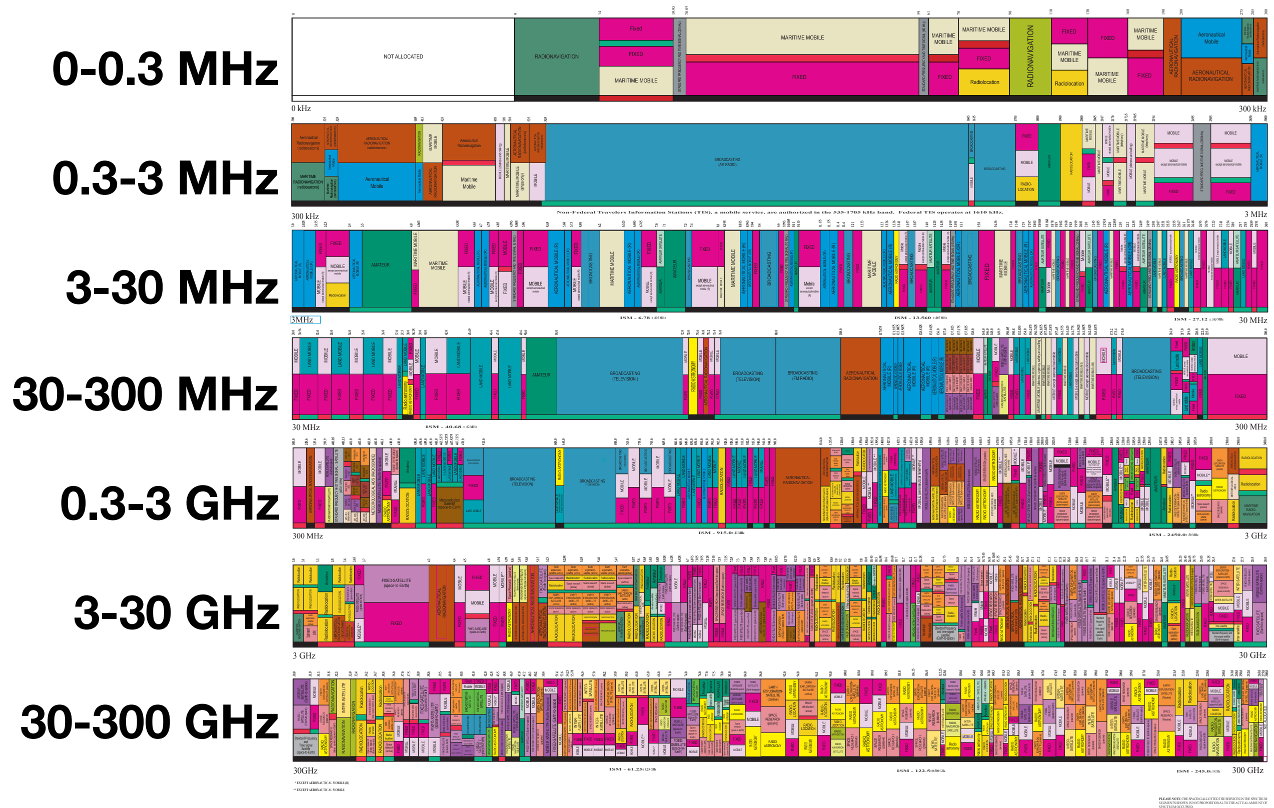
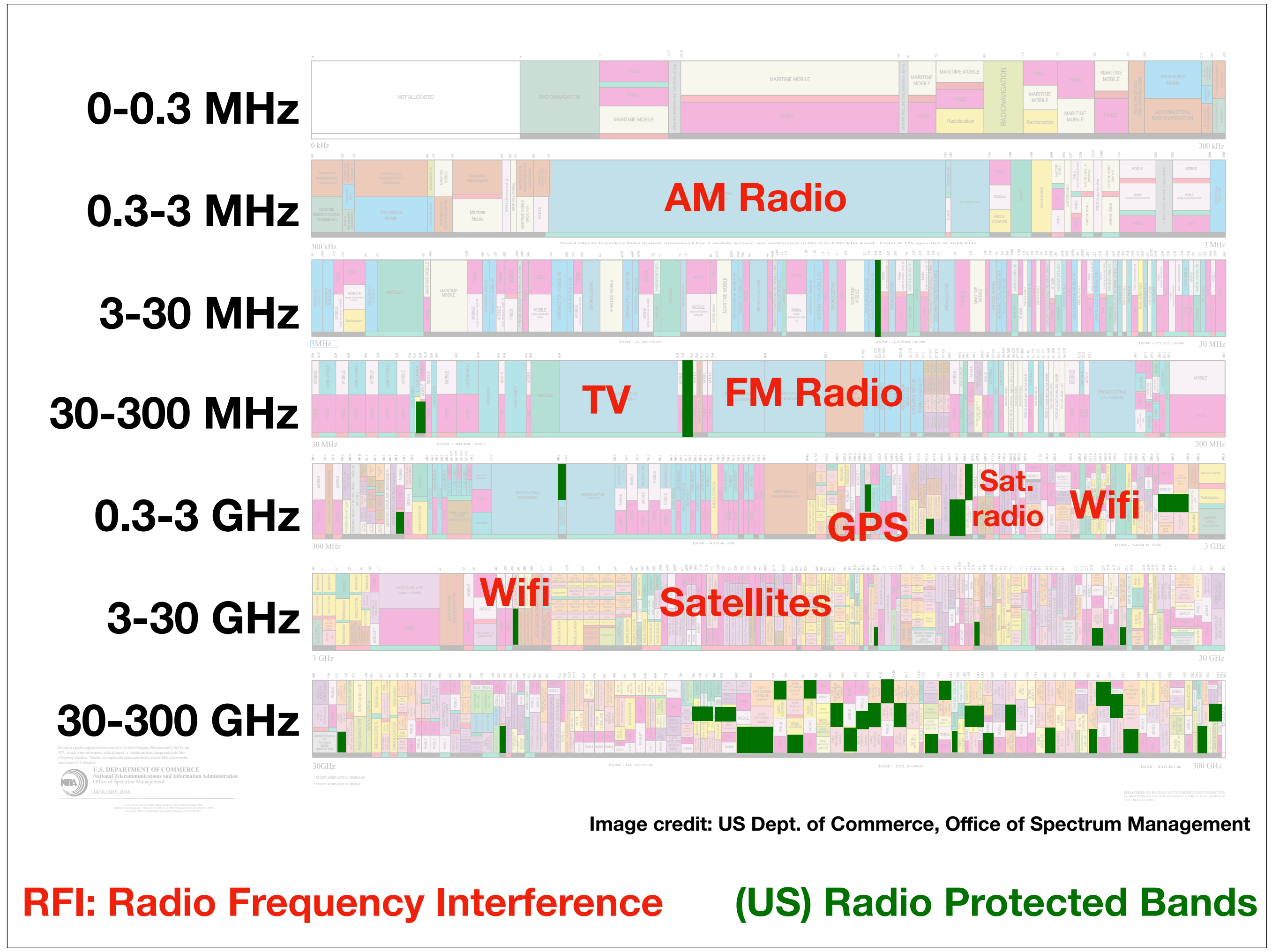
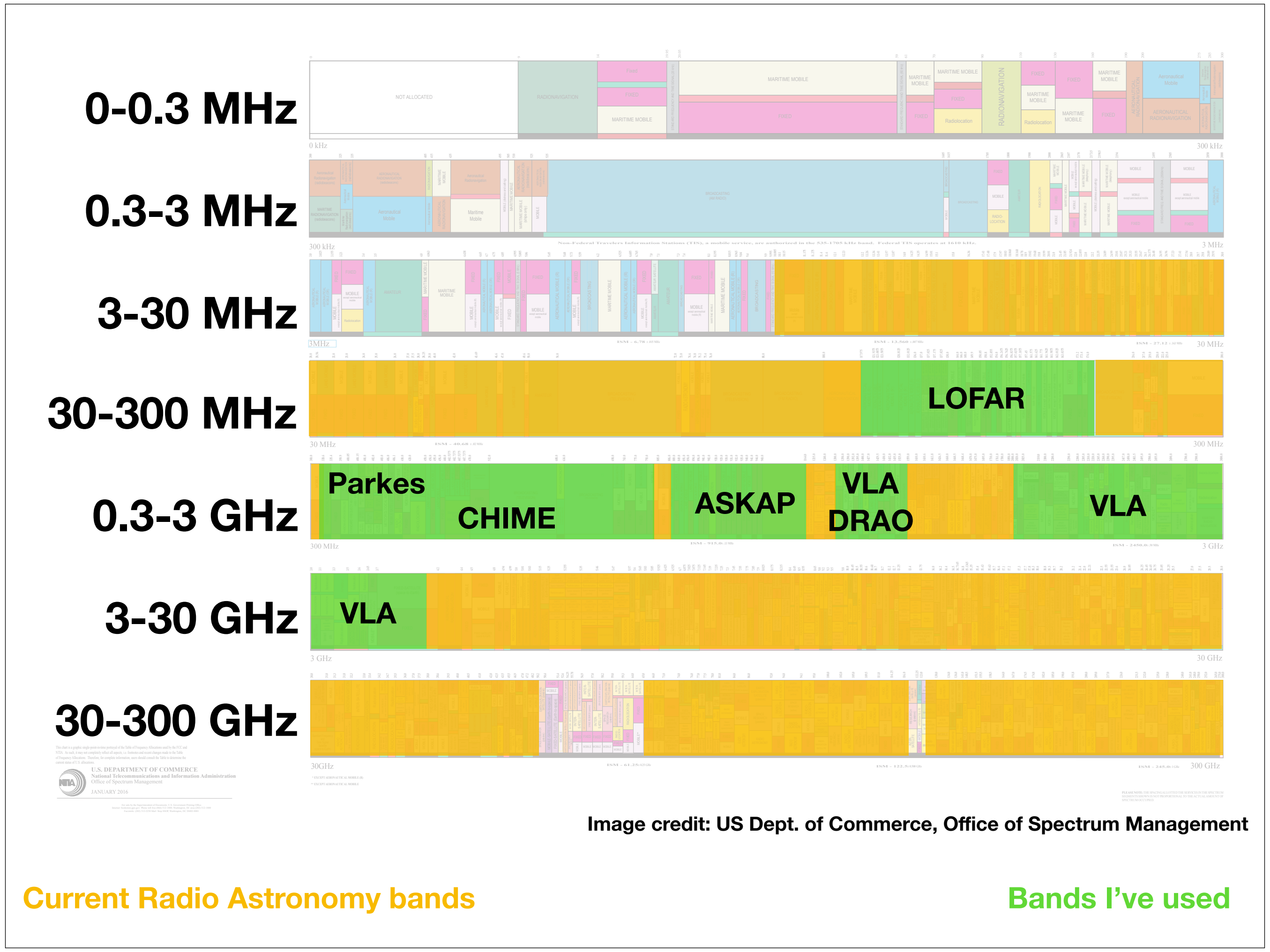


Image credit: US Dept. of Commerce, Office of Spectrum Management

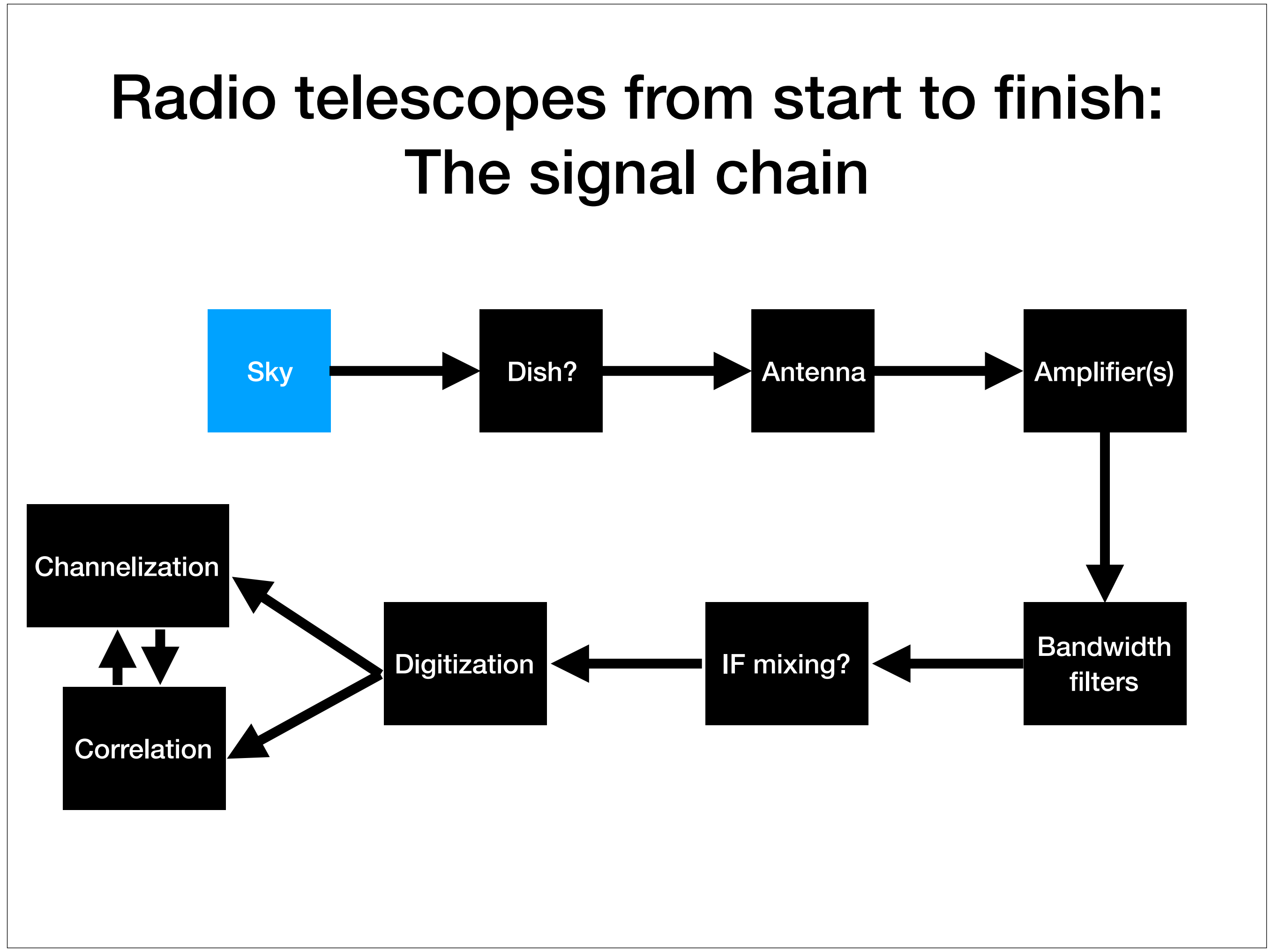
Here’s the frequency labels blown up to readable size. Each row is a decade in frequency/wavelength. Radio astronomy is the yellow boxes.



Highlighted the astronomy parts, as well as most common sources of interference.



Based on some quick searching, these are the parts of the spectrum for which I found at least one radio telescope.



An approximation of the steps involved in going from sky signal to telescope output. Not all telescopes will have all steps, or will do them in the same way.

EM waves

- Single monochromatic (linearly-polarized) plane wave:

$$\vec{E}(\vec{x},t) = \Re(\vec{E_0} e^{i(\vec{k}\cdot\vec{x} - 2\pi\nu t + \phi)}) \qquad \vec{E} = \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix}$$

- Treat the sky as a combination of such, integrating over direction and frequency:

$$\vec{E}_{\mathrm{sky}} = \int_{\mathrm{sky}} d\Omega \int_0^\infty d\nu \vec{E}(\vec{x},t)$$

Working in frame of reference where z is along propagation (\vec{k}) direction.
The linearly polarization isn't a requirement, it just makes it easier to write. Sometimes the circular basis is more useful…

Quasi-Monochromatic Radiation

- Analysis is simplest if the fields are perfectly monochromatic.
- This is not possible – a perfectly monochromatic electric field would both have no power ($\Delta\nu = 0$), and would last forever.
- So we consider instead 'quasi-monochromatic' radiation, where the bandwidth $\delta\nu$ is very small.
- For a time $dt \sim 1/d\nu$, the electric fields will be sinusoidal.
- Consider then the electric fields from a small sold angle $d\Omega$ about some direction **s**, within some small bandwidth $d\nu$, at frequency ν.
- We can write the temporal dependence of this field as:

$$E_\upsilon(t) = A\cos(2\pi\upsilon t + \phi)$$

- The amplitude and phase remains unchanged to a time duration of order $dt \sim 1/d\nu$, after which new values of **A** and ϕ are needed.

Image credit: Rick Perley (NRAO), 14th Synthesis Imaging workshop lectures

Jones formalism

$$\vec{E}(\vec{x},t) = \Re(\vec{E_0} e^{i(\vec{k}\cdot\vec{x} - 2\pi\nu t + \phi)}) \qquad \vec{E} = \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix}$$

$$\tilde{\mathbf{E}} = \begin{bmatrix} \tilde{E}_x \\ \tilde{E}_y \end{bmatrix} = \vec{E}_0 e^{i\phi}$$

- Phase, amplitude, polarization state of a monochromatic plane wave can be described by 4 parameters (2 complex numbers).

- Phase is relative to arbitrary reference point: one parameter can be dropped.

Also works in circular polarization basis!

Jones formalism

$$\tilde{E}_{\text{output}} = \mathbf{J}\tilde{E}_{\text{input}}$$

$$\mathbf{J} = \begin{bmatrix} a+bi & c+di \\ e+fi & g+hi \end{bmatrix}$$

- Linear operations to the signal become matrices.
- Circular <-> linear conversion is also a matrix.
- Polarization-independent effects are diagonal matrices.

This will come up a lot later. It's not a day-to-day tool, but it's a useful framework for understanding how things work under-the-hood.
Another reason this is really useful: it works for any wave-like signals, including AC electrical signals (so it can capture the sky to voltage conversion).

Flux and Intensity units

- Intensity/specific intensity/spectral intensity/spectral brightness, I_ν:

  Energy, per unit time, per unit frequency, per unit collecting area, per unit solid angle

  Independent of instrument, independent of distance.

  Same units as Planck equation.

- Integrating over solid angle of a source gives flux density, S_ν.

- Integrating flux density over frequency gives flux, S.

Flux density is often called just ‘flux’, because we rarely integrate over frequency. I make this mistake a lot, so call me out on it! I also screw up by using intensity and flux interchangeably.

Flux and Intensity units

- Radio unit of flux density: the Jansky.
- 1 Jansky (Jy) = 10^{-26} W m^{-2} Hz^{-1}
- Specific intensity often in Jy/beam, Jy/sr, MJy/sr
  Or in Kelvin (more on that later)

Antennas

- A device that converts EM radiation into electric currents (or vice versa). I.e., a conductor attached to a circuit.
- Fun fact: the plural of radio antenna is antennas, plural of insect antenna is antennae.

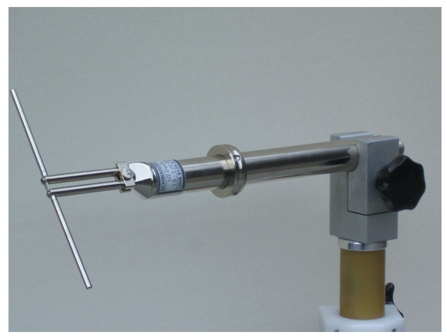


Image credit: Schwarzbeck Mess-Elektronik, Wikimedia Commons

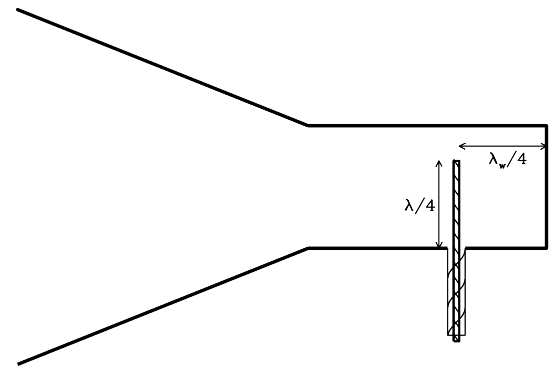


Image credit: Fig 3.3 from Essential Radio Astronomy

Also: Any given antenna is generally only sensitive to one polarization of light (either linear or circular). Need two antennas to get full sensitivity to polarization state.

Image credit: Apostolos Spanakis-Misirlis

Image credit: ASTRON

Image credit: ASTRON

Image credit: Wikimedia Commons User Kingbastard

Top left: ‘PICTOR’ amateur ratio telescope. 21cm cylindrical feed, copper 1/4 wavelength antenna clearly visible.

Top right: LOFAR low-band antennas

Bottom left: LOFAR high-band antennas, ‘butterfly dipoles’

Bottom right: Satellite-finding radar with helical antennas

Antenna beams

- ‘beam’: 2D pattern of sensitivity as a function of direction
- Possibly the most overused/vague term in radio astronomy: antenna beam, primary beam, synthesized beam, dirty beam, clean beam, element beam, station beam, etc.
- Specific usage depends on context: make sure you know which one is being discussed!

Antenna beams

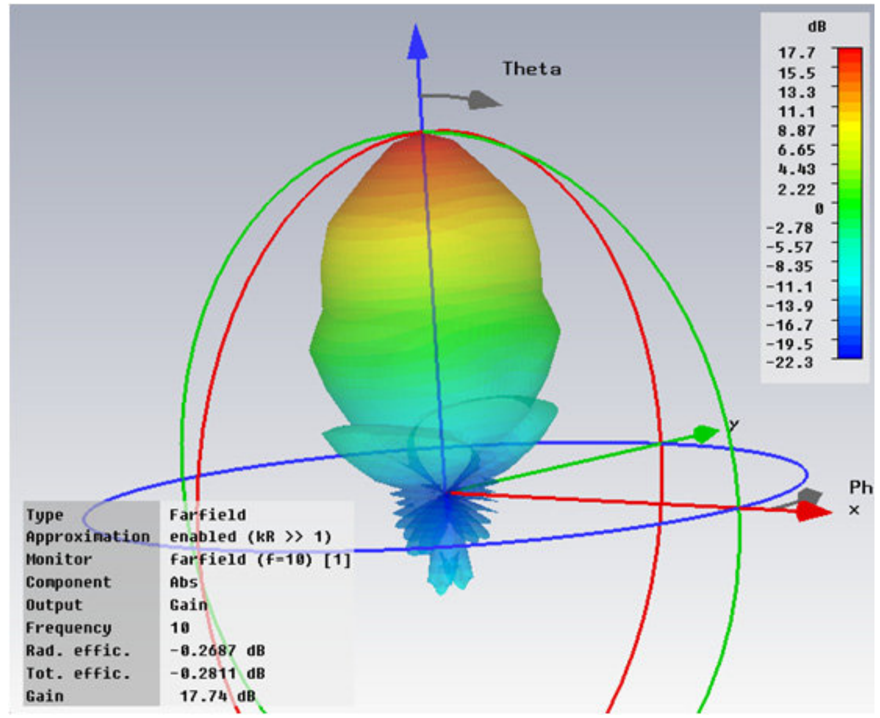


Image credit: Duangtang et al 2016

Beam pattern of some antenna. Distance from the origin, and color, are proportional to sensitivity.

Antenna beams

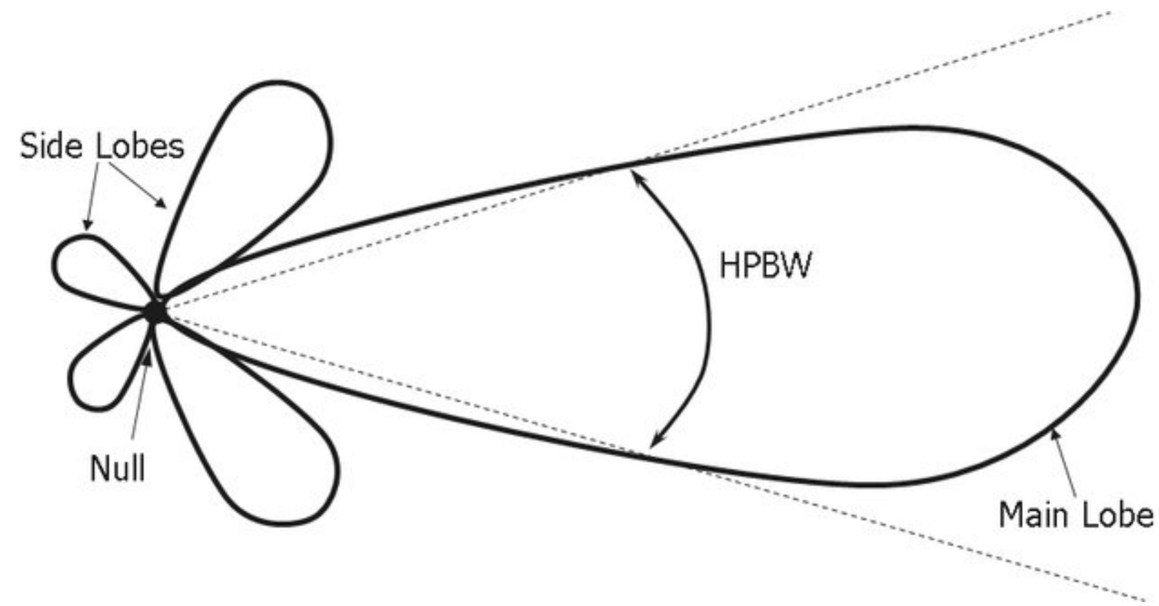


Image credit: Babu et al 2011, 10.1109/ISPACS.2011.6146213

2D cross section. Half-power beam width (HPBW) is the most common definition for the ‘size’ of the beam.
First null is also sometimes discussed.
Skipping over details of gain and equivalent area and such. Not particularly important.

Antenna power units

- The antenna converts electric field of EM wave into current.
- Current passing through circuit has some power.
- Power can be interpreted in temperature units.

Power produced by the antenna is the power of the wave, minus any signal not absorbed, or re-radiated.

Antenna power units

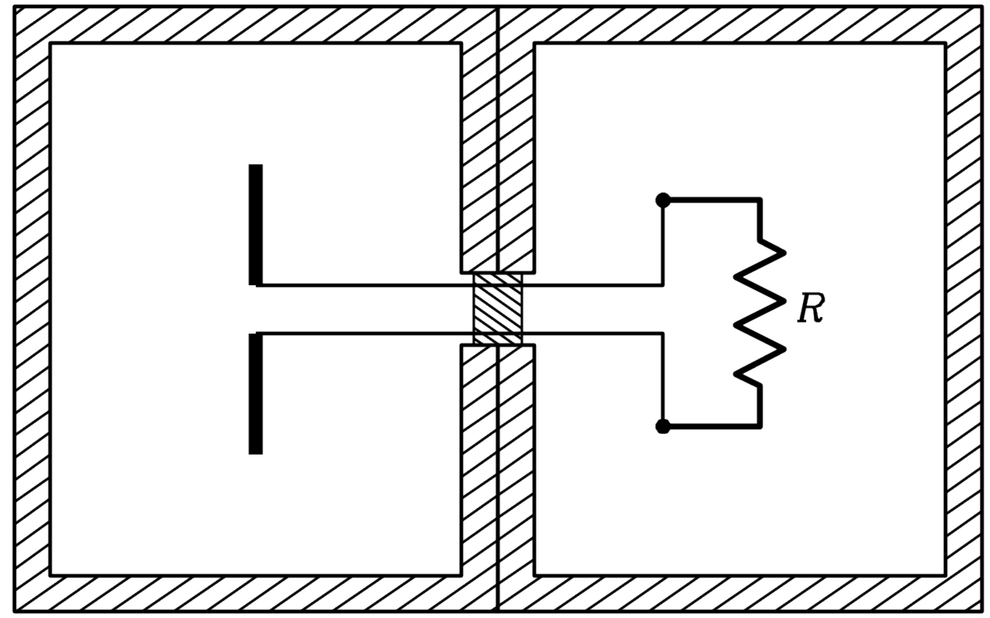
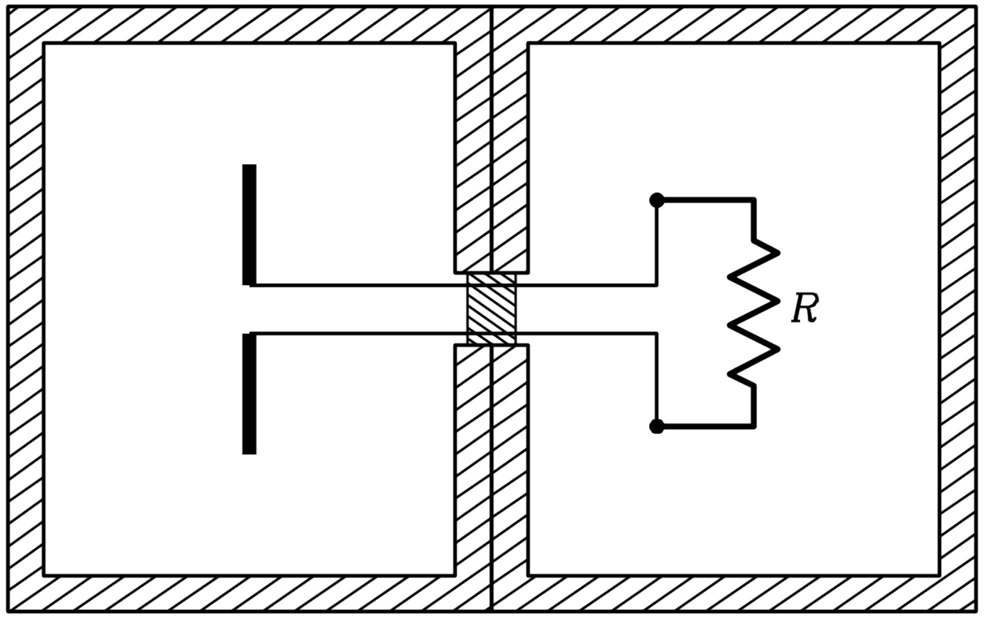


Image credit: Fig 3.5 from Essential Radio Astronomy

Blackbody radiation power enters antenna, gets dissipated by resistor.
Can’t violate laws of thermodynamics: can’t concentrate energy from one side to another.
Therefore, resistor sends signal to antenna, which re-radiates.
Equilibrium: power entering an antenna corresponds to some equivalent temperature for the resistor.

Antenna power units



Johnson-Nyquist noise: $P_\nu = kT_A$

Image credit: Fig 3.5 from Essential Radio Astronomy

Power per frequency = Boltzmann constant times antenna temperature.
Units: Power per frequency = Energy per time per (inverse time). k has units of energy per temperature.

Temperature units

- Antenna temperature: temperature of blackbody that has equivalent power per frequency at a particular frequency to what the antenna is receiving.
- Brightness temperature: temperature of a blackbody that produces the same **intensity** at a particular frequency.
- In general, invert Planck equation to get temperature.
- In radio, everything is in the Rayleigh-Jeans limit: $T_b = \frac{I_\nu c^2}{2k\nu^2}$
- If a source fills the beam: $T_A = T_b$
- Can always convert between intensity (I_ν) and T_b

Dishes

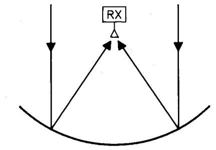
- Are basically just optical components, exactly like traditional optical telescopes.
- Longer wavelengths means diffraction effects are much stronger (scale as λ/D).
- Surface accuracy also scales ($\sim\lambda/10$), so less polishing needed.

The much larger λ/D causes wave effects to be significant: can't really model things in terms of ray optics; everything is wave optics.

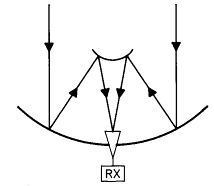
Reflector antennas

GMRT

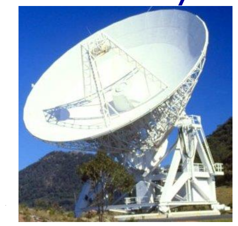
Prime Focus



On-axis Cassegrain



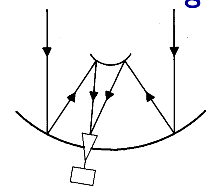
(best for array receivers)



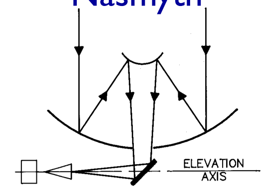
ATCA, Mopra

VLA, ALMA

Offset Cassegrain



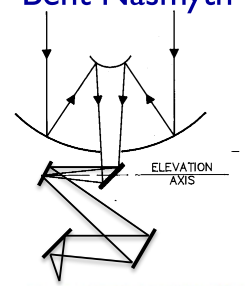
Nasmyth



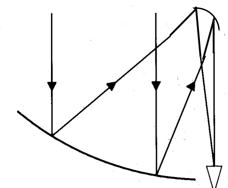
CARMA, CSO

SMA

Bent Nasmyth



Dual offset Gregorian



GBT

Receivers do not tilt in elev.

Cleanest beam, minimizes standing waves, polarization asymmetry compensated -- Mizugutch et al. (1976)

Image credit: Todd Hunter (NRAO), 14th Synthesis Imaging workshop lectures

The Parabolic Reflector

- Key Point: Distance from incoming phase front to focal point is the same for all rays.
- The E-fields will thus all be in phase at the focus – the place for the receiver.

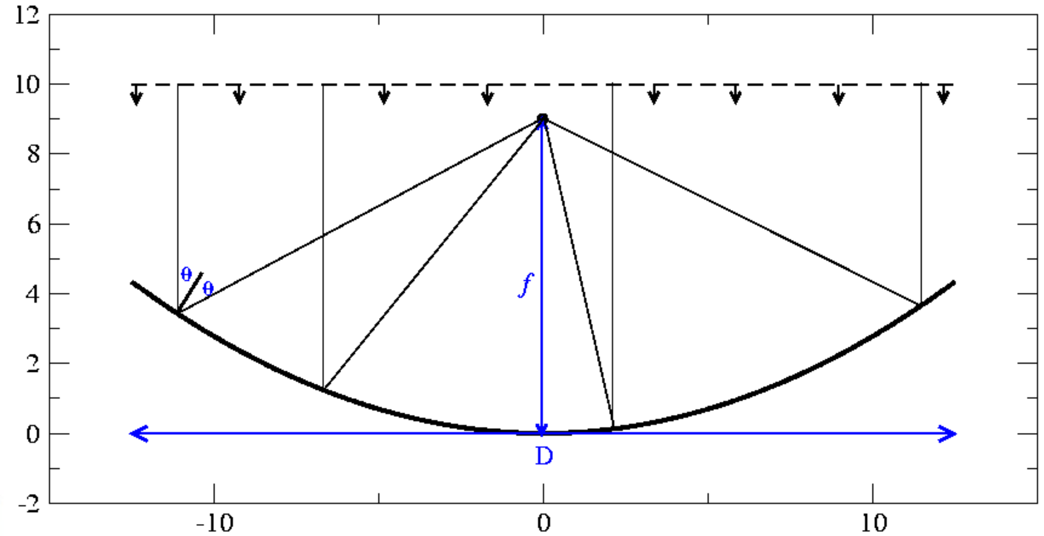


Image credit: Rick Perley (NRAO), 14th Synthesis Imaging workshop lectures

Beam Pattern Origin
(1-Dimensional Example)

- An antenna's response is a result of coherent vector summation of the electric field at the focus.
- First null will occur at the angle where one extra wavelength of path is added across the full width of the aperture:

$\theta \sim \lambda/D$

(Why?)

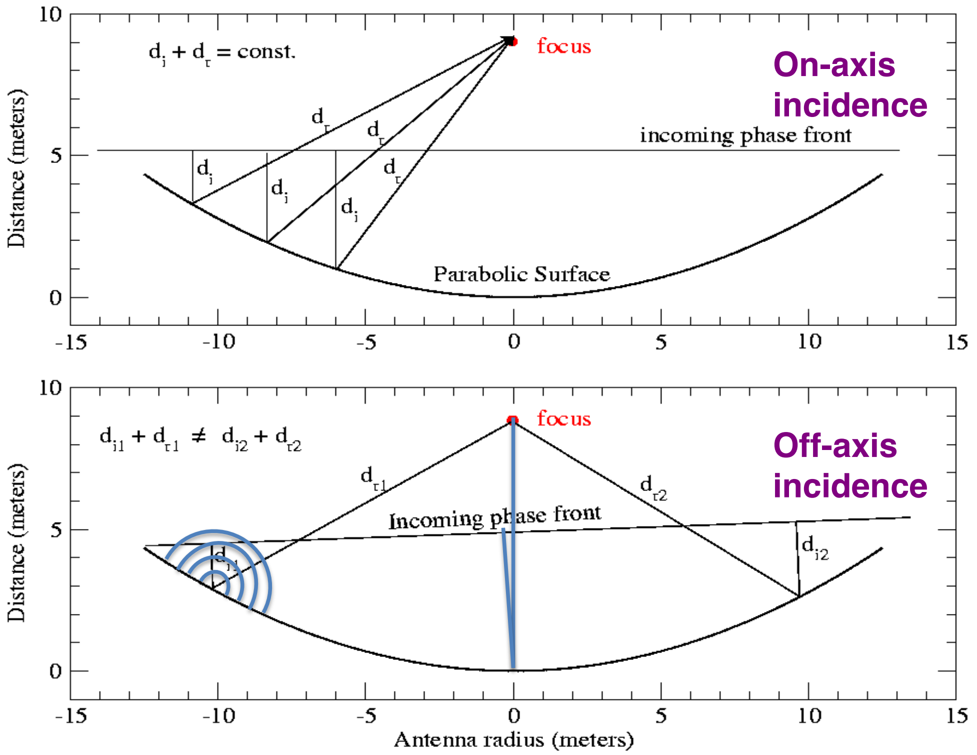


Image credit: Rick Perley (NRAO), 14th Synthesis Imaging workshop lectures

Wave optics, Huygens-Fresnel principle, dominate off-axis behaviour, leading to sidelobes

Dish beams

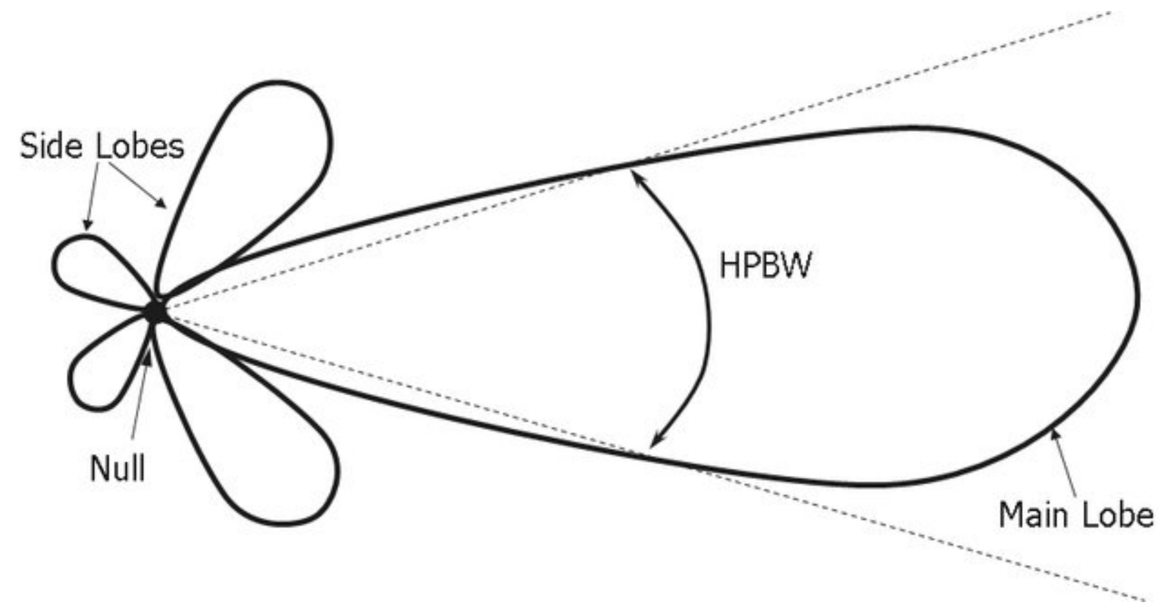


Image credit: Babu et al 2011, 10.1109/ISPACS.2011.6146213

2D cross section. Half-power beam width (HPBW) is the most common definition for the ‘size’ of the beam.
First null is also sometimes discussed.
This will be modified by presence of support structures for the secondary.
Beam is intrinsically complex quantity (even though only amplitude is plotted): phase indicates the phase-shift applied incoming signal.
Skipping over details of gain and equivalent area and such. Not particularly important.

Phased Arrays
~~Interferometry~~ – Basic Concept

- We don't need a single parabolic structure.
- We can consider a series of small antennas, whose individual signals are summed in a network.
- This is the basic concept of interferometry.
- Aperture Synthesis is an extension of this concept.

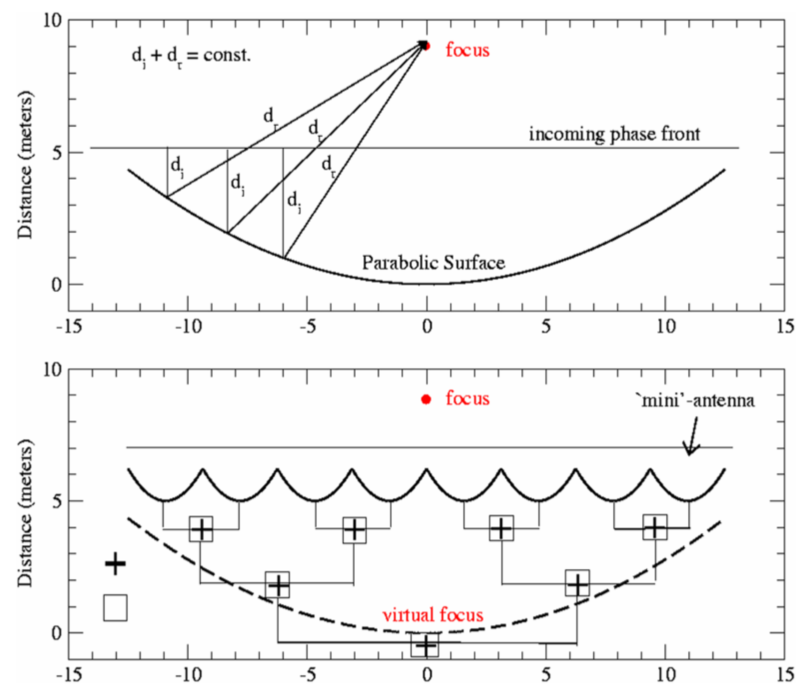


Image credit: Rick Perley (NRAO), 14th Synthesis Imaging workshop lectures

Phased Arrays

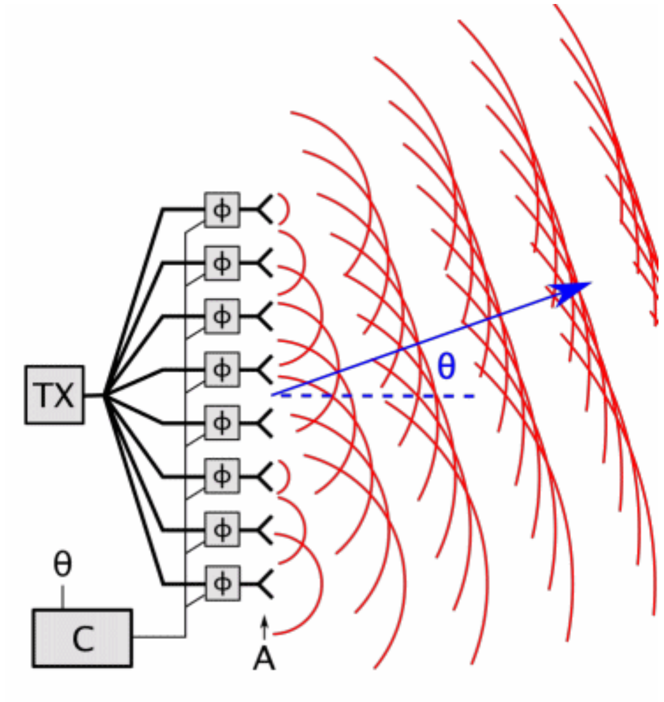


Image credit: Wikimedia commons

Phased Arrays

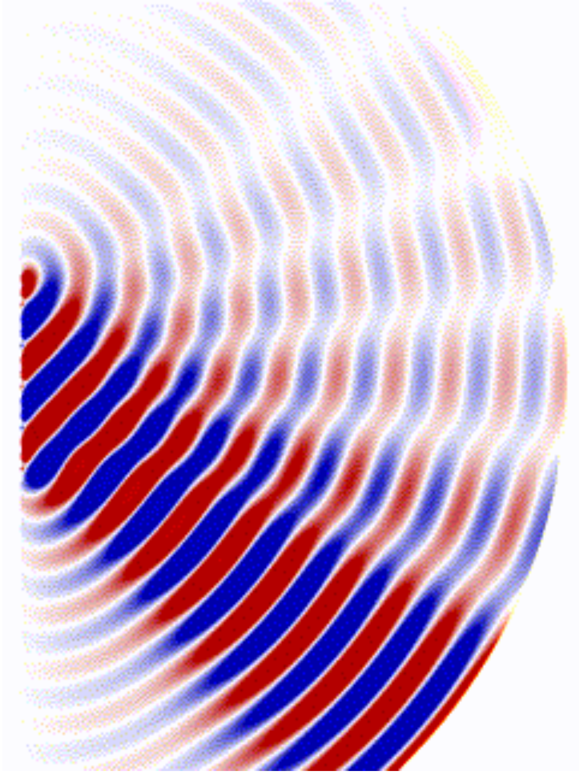


Image credit: Wikimedia commons

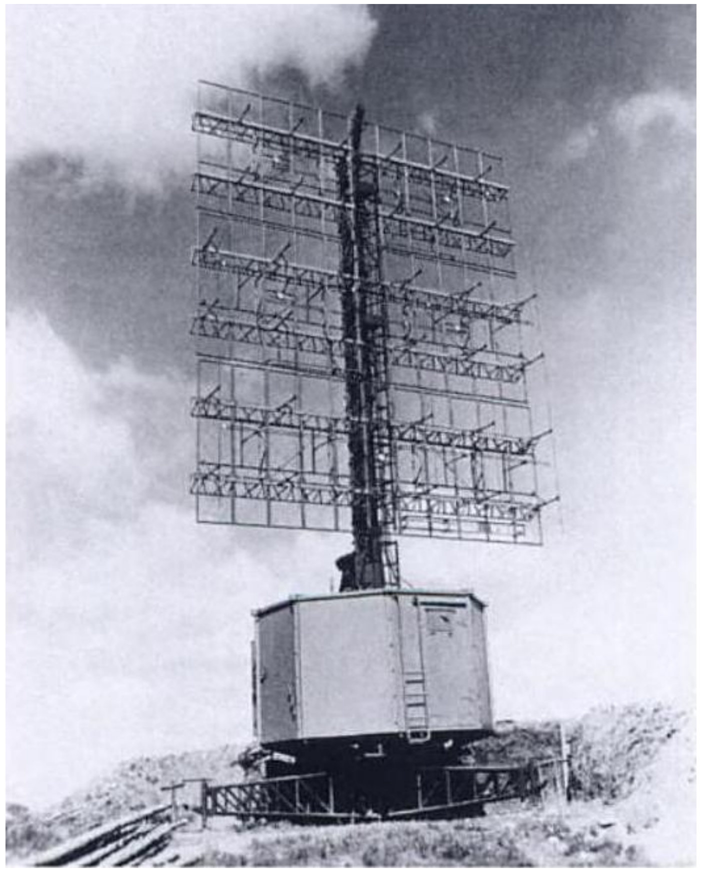
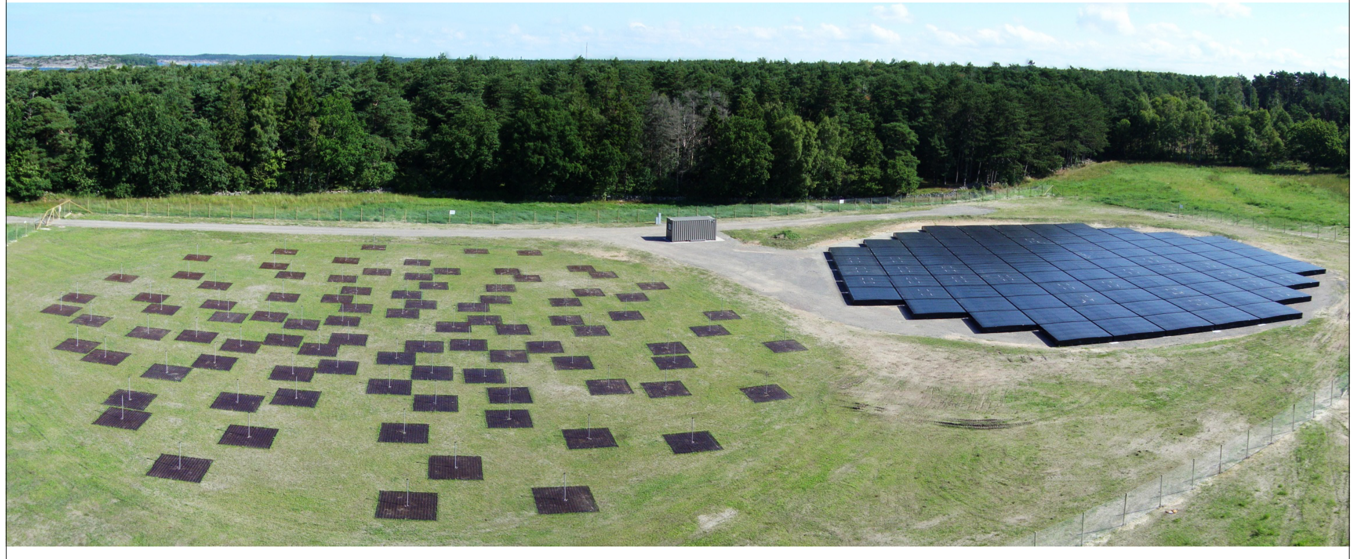


Image credit: US National Archives and Records Administration - Foto 111-SC 269043

Phased arrays are almost as old as dishes: this is a WWII radar station.



A LOFAR station: two phased arrays at different frequencies. The 96 low-band antennas on the left operate as one phased array, while each of the square tiles on the right contains 16 high-band antennas that are phased together, and then each of the 64 tiles are phased together.

Beam patterns

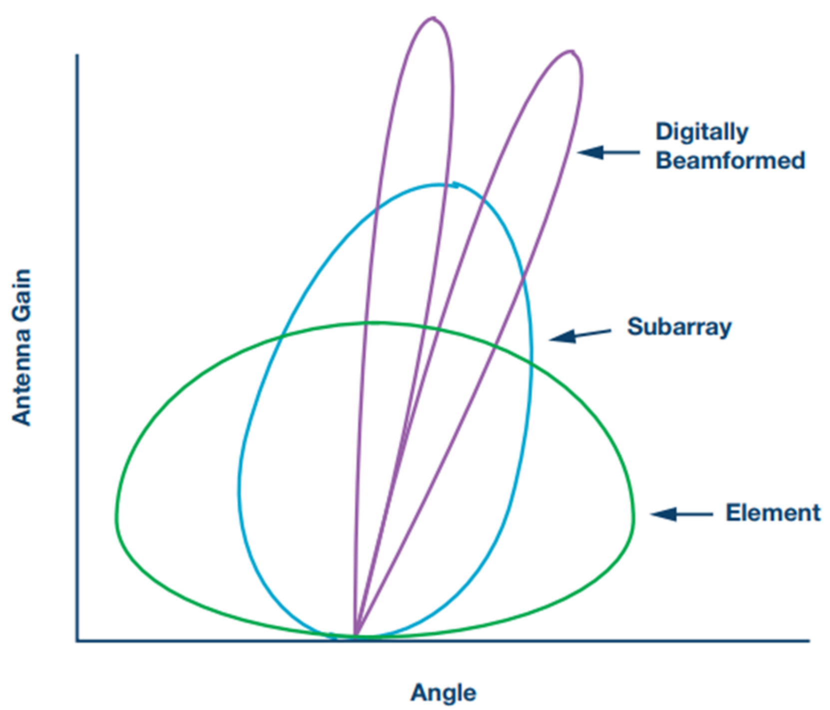


Image credit: Peter Delos, Analog Devices

Each ‘layer’ in a phased array has it’s own beam, determined from the (complex) summation of beams of the previous layer. This starts with the antenna/element beam, and builds up depending on how the signals are added.

In this plot, the phased array has 3 layers: individual antennas/elements, ‘subarrays’ made of several antennas phased together, then it is implied at the signal is digitized and the then subarrays are phased together.

Beams as Jones matrices

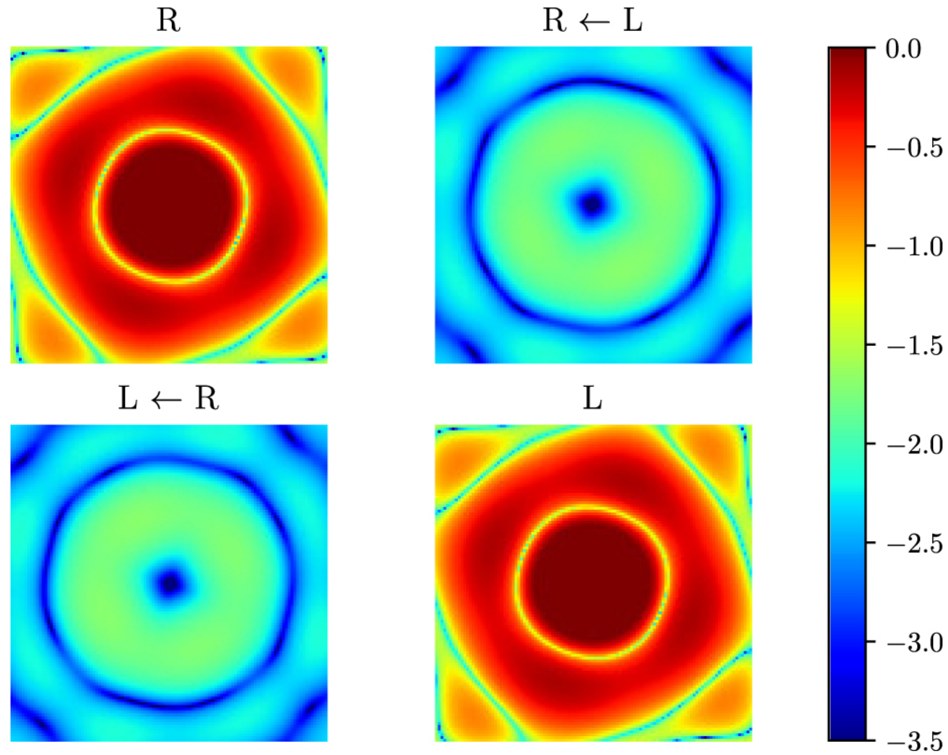


Image credit: Jagannathan et al 2017, ApJ 154

The previous beams were just about power/signal strength, but the effect of the dish/optical elements/phasing can be more complicated, introducing phase shifts and being polarization-dependent.
This can all be described as a position-dependent Jones matrix

Note the 4 sided support structure! This leads to the square shape of the beams seen on the last slide.

Geometric delay

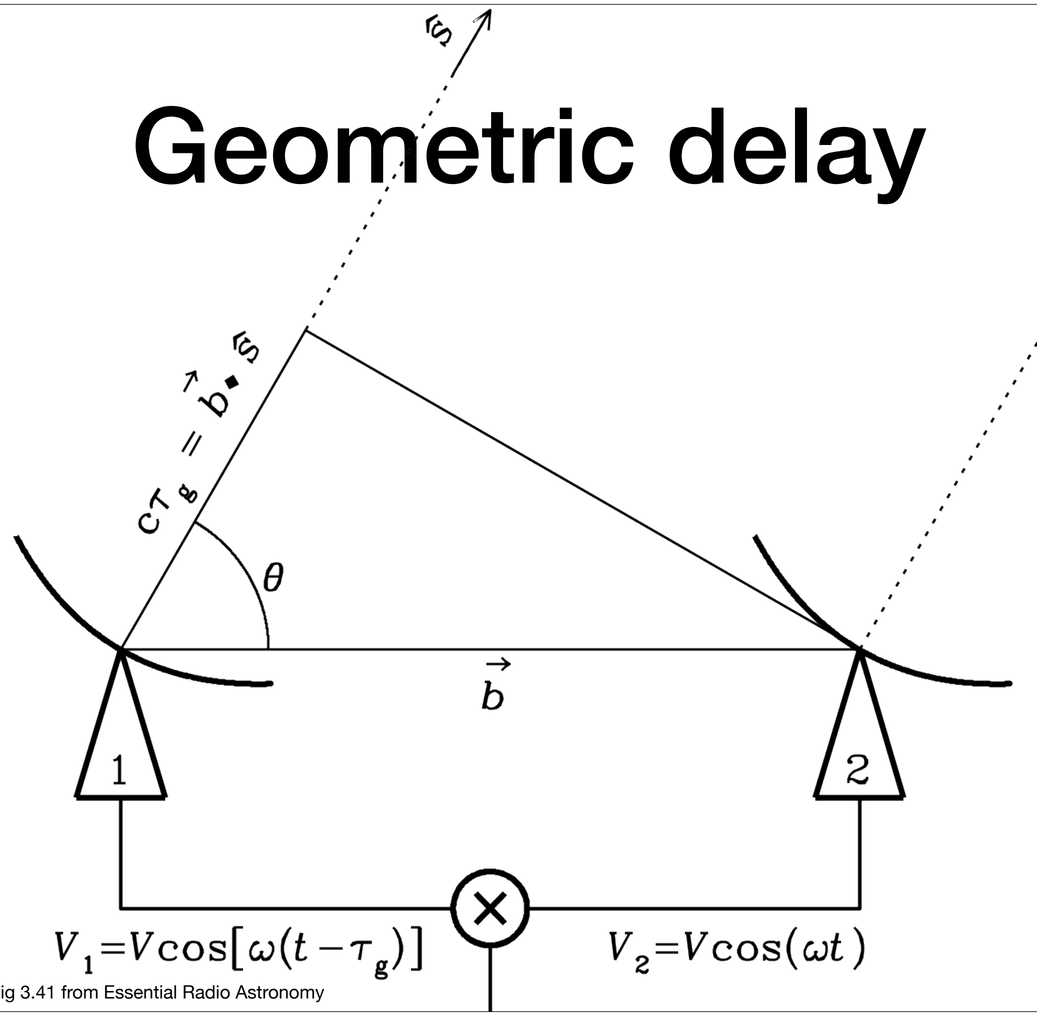


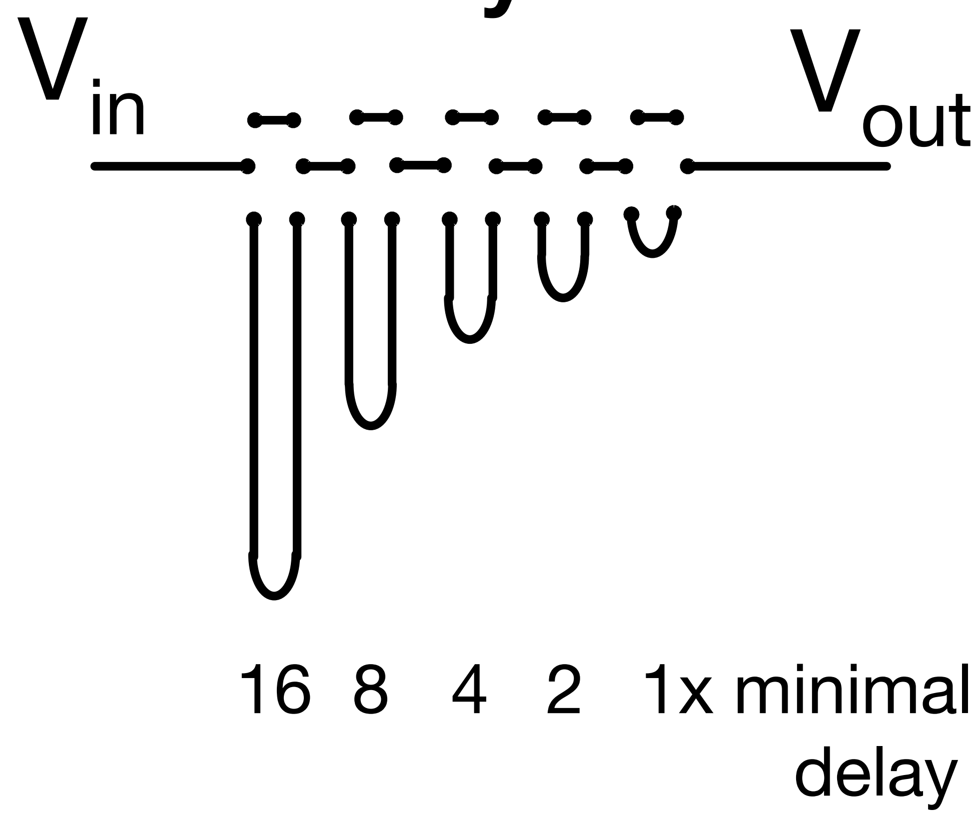
Image credit: Fig 3.41 from Essential Radio Astronomy

Matters mostly for interferometers (since baselines tend to be much longer), but may be relevant for phased arrays, and the physics is the same. Geometric delay is the difference in arrival time of the wave at the two elements being considered.

Geometric Delay

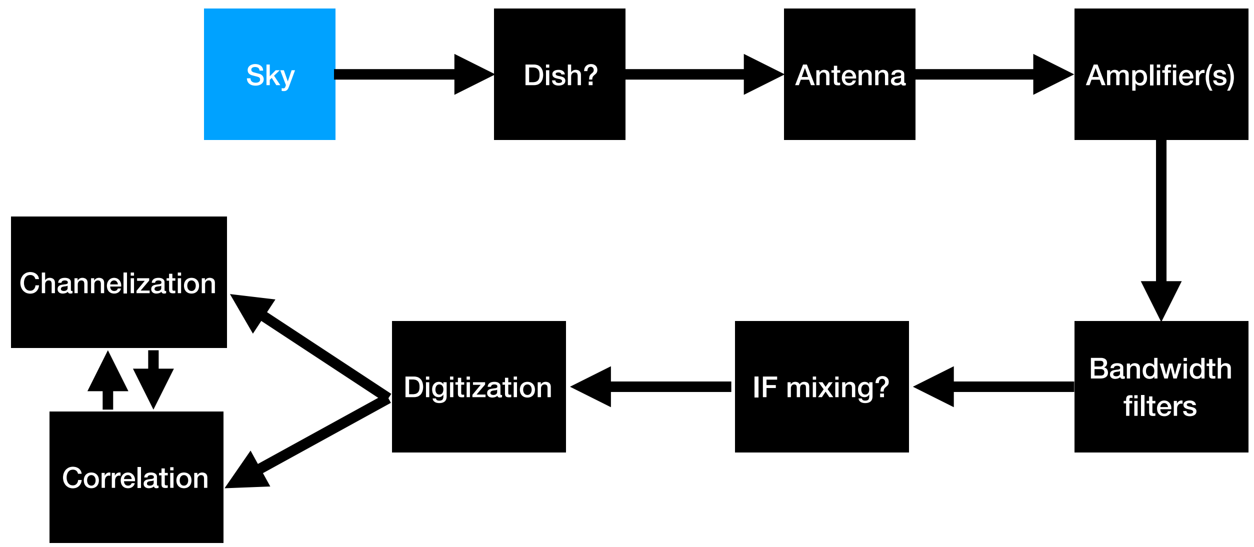
- Recall quasi-monochromatic approximation: wave parameters only constant in time frame of $dt \sim 1/d\nu$
- Waves with different parameters will not yield any correlation/will not phase-up.
- Therefore, geometric time delay must be accounted for before combining signals.
- Use delay lines: force signals from nearer antenna to go through cable of appropriate length; known speed of electricity can give same delay as for light propagation.

Delay line



One type of delay line system: the input signal can go through different lengths of wire, depending on the switches are set. Lengths of each line are a power of 2, allowing the system to set any multiple of the minimal delay. This ensures the delay is corrected to within half the minimal delay, which can be made smaller than dt~1/dv.
In digital systems (where signals are digitized before correlation/addition), this can be done with memory buffers.

Radio telescopes from start to finish: The signal chain



An approximation of the steps involved in going from sky signal to telescope output. Not all telescopes will have all steps, or will do them in the same way.

Amplifiers

- Amplifiers increase signal strength, to minimize effects of noise from later components in the chain.
- Also amplify any noise present before the amplifier (plus any noise introduced by the amplifier). Essential that signals are amplified as early in signal chain as possible.
- Amplifiers must be intrinsically linear ($V_{out} = A*V_{in}$); input signals that are too strong can put amplifier in non-linear regime (saturation).
- Amplifier gain can be frequency dependent: output signal strength has artificial frequency structure that must be calibrated out.

Saturation is a big problem because it breaks calibration. VLA has had problems with saturation from satellites, had to step down amplifiers causing lower sensitivity. Correcting for instrumental frequency structure is bandpass calibration.

Bandpass filters

- Filters that selectively pass or damp out certain frequencies; used to remove non-desired signals to prevent them from affecting down-stream components.
- By design, affect frequency response; even in the desired pass-band the response will not be flat (passing all frequencies equally).

Removing strong signals that might saturate amplifiers is very desirable.

Heterodyning

- Traditionally, high frequency electronics are hard to build. (Commodore 64 had 1 MHz processor, in 1982. My first computer, an 80486, was ~20 MHz in 1992). Even now, going above a few GHz is hard.

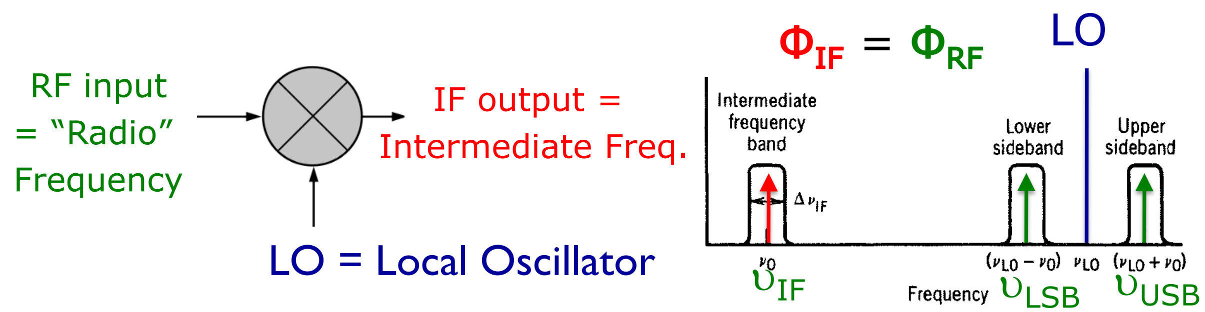
- Digitization, in particular, needs electronics that run at twice the highest frequency being measured (Nyquist sampling).

- Signals can be down-converted in frequency, by mixing (multiplying) them by a signal at a different frequency, producing outputs at an intermediate frequency.

What is a mixer?

Mixers are 3-port devices: LO and RF inputs, and IF output.

- Invented around WWI for radio direction finding (see IEEE Microwave Magazine Sept. 2013 special issue).
- They multiply the LO & RF signals and transfer the phase from the RF to the IF by "heterodyning". Typically the IF contains signals from two sidebands.
- They are key components for interferometers!!



$$\sin(2\pi f_1 t)\sin(2\pi f_2 t) = \frac{1}{2}\cos[2\pi(f_1 - f_2)t] - \frac{1}{2}\cos[2\pi(f_1 + f_2)t]$$

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Image credit: Todd Hunter (NRAO), 14th Synthesis Imaging workshop lectures

'Heterodyne': making signal with a new frequency by combining/mixing two signals with different frequencies.
Side effect of mapping two 'sidebands' onto the same frequency range. Various solutions to this, but not interesting enough for this course.

Digitization

- At some point, we need to convert the (analog) voltage signals into digital signals.
- Analog-digital converter (ADC) accomplishes this. Not much to be said about these, except the bit depth: how many bits are used to represent each measurement.
- Since signals are almost immediately Fourier transformed, correlated, and/or averaged, often not many bits are needed.
- Digital information is less vulnerable to noise, can be duplicated as much as needed -> this allows digital beam-forming phased arrays to have many simultaneous beams

Traditionally, digitization can take place almost arbitrarily late in the process, but with modern computer systems it’s often advantageous to do it as early as possible, because computers are cheap.

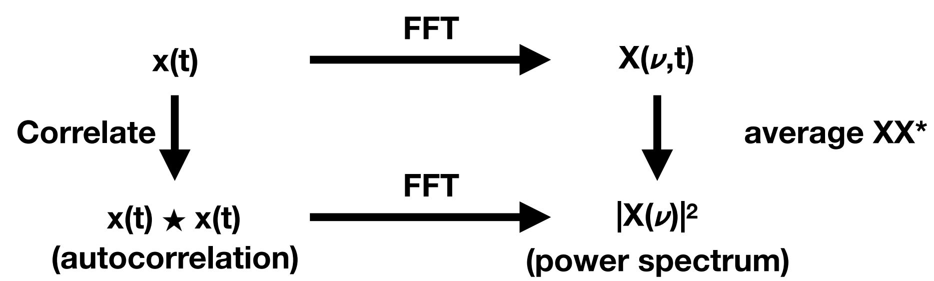
Channelization/ Fourier transform

- Raw voltage time series is rarely interesting/useful.
- Signal vs frequency (Fourier transform) is much more interesting (plus we need to calibrate for bandpass effects, which work in frequency-space).
- Uses FFT algorithm: converts N (real) time-samples into N (complex) channels. Produces a spectrum every $N/(2\Delta\nu)$ seconds, which can then be (power-) averaged or otherwise processed as desired.

FFT = Fast Fourier transform, which is most efficient algorithm (O(N log N))
Delta-nu is bandwidth being sampled; factor of 2 is due to Nyquist sampling theorem.
Can't average complex numbers directly: quasi-monochromatic approximation means that phases randomize over some time period. So averaging is done in power (amplitude squared).

Channelization

- Alternative method: compute autocorrelation of time-series, then FFT to get averaged power spectrum.



- Computing a power spectrum loses phase (and polarization) information. To get full polarization, need two antennas and compute 2 autocorrelations and 2 cross correlations (equivalent to XX*, YY*, XY*, YX*).

X(v,t) is series of spectra (one spectrum every N points).

Radiometer equation

- Condon and Ransom have a more physically motivated derivation; my version is more mathematical.
- Consider a 1-channel system, integrating the power in some bandwidth $\Delta\nu$.
- All the signal (voltage) sources in the system (thermal noise, amplifier noise, sky signals (T_A)) are treated as Gaussian (zero mean, some σ_V). Sum of Gaussian processes is Gaussian (central limit theorem). Note that sky may only be small contribution!
- Converting to power turns this into a χ^2-like distribution of order 1, with mean of σ_V^2 and variance $2\sigma_V^2$.

σ_V is rms voltage of combined noise signals.

Radiometer equation

- Converting voltage to power turns this into a χ^2-like distribution of order 1, with mean of σ_V^2 and variance $2\sigma_V^4$.
- Power (per unit frequency) can be converted to temperature units: $P_\nu = kT_s$ has units of power per frequency
- System temperature: $T_s = \sigma_V^2 / (k\,\Delta\nu)$ is (sky) temperature producing power equivalent to telescope output signal.
- Note: Antenna temperature is power absorbed by antenna. System temperature is power output by signal chain.

Ts is system temperature, the equivalent temperature of the all signals.

Radiometer equation

- System temperature: $T_s = \sigma_V^2 / (k\ \Delta\nu)$ is (sky) temperature producing power equivalent to telescope output signal.

- Instantaneous uncertainty in T_s: $\sigma_T = 2^{1/2}\ \sigma_V^2 / (k\ \Delta\nu) = 2^{1/2}\ T_s$

- Average together N independent measurements to reduce uncertainty. Note quasi-monochromatic condition: signals within $\Delta t \approx 1/\Delta\nu$ are not independent. So an observation of length τ has $N = \tau / \Delta t = \tau\ \Delta\nu$.*

- Uncertainty drops as $1/N^{1/2}$: $\sigma_T \approx (\sqrt{2}?)\dfrac{T_s}{\sqrt{\tau \Delta\nu}}$

*: All sources I find tack on a factor of 2 here, but don't explain why very well.

Radiometer equation

$$\sigma_T \approx (\sqrt{2}?)\frac{T_s}{\sqrt{\tau \Delta\nu}}$$

- Isolating sky signal (T_A) requires subtracting off system (noise) contributions to T_{sys} (as determined by calibration). Thus, S/N in sky signal depends on S/N in T_{sys}: more time and more bandwidth improve S/N.

- Typical values for $\boldsymbol{\tau}\ \Delta\boldsymbol{\nu}$ can be in excess of 10^8, so even weak signals can be isolated. Eventually, this equation breaks when systematic errors start to take over.

Examples of systematic errors include calibration errors, variations in system properties over time (e.g. gain).

Jones matrices for signal chain?

Hamaker et al. 1996:

The equation describing the signal path in antenna A is then

$$\boldsymbol{v}_A = \boldsymbol{J}_A \boldsymbol{e}_A \; ; \quad \boldsymbol{J}_A = \boldsymbol{R}_A \boldsymbol{C}_A \boldsymbol{P}_A \boldsymbol{F}_A \tag{11}$$

Remember that the Jones matrix is for a matched pair of dual-polarization receiver systems.
e_A = (sky) electric field, v_A = output voltage
J_A = Jones matrix representing everything antenna/telescope A does to the signal

Jones matrices for full telescope

Hamaker et al. 1996:

The equation describing the signal path in antenna A is then

$$\boldsymbol{v}_{\mathrm{A}} = \boldsymbol{J}_{\mathrm{A}} \boldsymbol{e}_{\mathrm{A}} \; ; \quad \boldsymbol{J}_{\mathrm{A}} = \boldsymbol{R}_{\mathrm{A}} \boldsymbol{C}_{\mathrm{A}} \boldsymbol{P}_{\mathrm{A}} \boldsymbol{F}_{\mathrm{A}} \qquad (11)$$

- F_A = Faraday rotation (by ionosphere)
- P_A = Parallactic angle (rotation between sky frame and instrument frame)
- C_A = Effects of the feed (dish + antenna) on the signal
- R_A = Effects of the receiver system (amplifiers, filters, etc)

F can be ignored: it can be treated as part of the astrophysical problem instead of the observation.
P is a straight forward rotation matrix (as a function of time), depending on the telescope mount
C encodes all the beam (position dependent) effects.
R encodes the effects of all the electronics: amplification, phase shifts, time delays, etc.