Exercise problem 1:
Source counts and radio brightness: how bright is the radio sky?

A professor of mine once told me that all of the radio energy collected by some radio survey adds up to less than the kinetic energy of a falling snowflake. Is that true? Let's try to reproduce the calculation. We'll calculate the numbers for one of the bigger radio surveys: the NRAO VLA Sky Survey (NVSS). How much energy was collected by the NVSS? Let's do an order-of-magnitude estimate.

1. Look up key properties of the NVSS. In particular, we'll need to know the sky area of the survey, the frequency and bandwidth, the observation time (time spent observing any particular chunk of sky), and the sensitivity. The main paper describing the NVSS survey is Condon et al. 1998.

2. Next, we'll need to know how much flux density comes from radio sources in the sky. The measurement of how many radio sources there are, as a function of flux density (S), is called source counts. This is given by n(S), where the number of sources with flux density in the range [S, S+dS] is n(S) dS. So n(S) has units of inverse flux density. For reasons relating to cosmology, many authors report it as n(S) multiplied by $S^{(5/2)}$. Source counts change with frequency (because different populations of radio sources have different spectra).

Conveniently, Condon et al. 1998 includes source counts calculated directly from the NVSS data, in Figure 1. We need n(S) from this figure for the next step. In order to do calculations with this, let's fit-by-eye a broken power law (straight lines) to the data. Break the data into 3 or so segments that each seem reasonably straight, estimate the slope and normalization of each segment, and write a piecewise function for n(S) (in sr^{-1} Jy^{-1}).

3. a) Since n(S) is the number of sources of flux density S, S*N(S) is the flux from all of those sources. Integrate that over all possible flux densities to see how much flux is coming from all sources, per unit solid angle. Extrapolate your broken power laws out to zero flux and infinite flux, and set the integral limits as zero to infinity.

b) Spoilers: the integral in (a) diverges. What does that mean, physically and for the model? What prediction can you make for how Condon+98's Fig 1 must look if we measured below 10^-4 Jy?

c) Let's try again, ignoring the faintest sources (integrate from 0.1 mJy to infinity).

4. We should now have all the values needs to determine the total energy collected. Remembering that 1Jy = 10^-26 W/m^2/Hz, using the sky flux per solid angle (from previous part), sky area of the survey, collecting area of telescope (look up the dimensions and number of dishes in the VLA), the observation duration, and the bandwidth, calculate the total energy imparted by extragalactic radio sources. Check that the units work out correctly!

5. Now, what is the kinetic energy of a falling snowflake? Go look up typical mass and terminal velocity of a snowflake and report back (with references). Google is your friend! Calculate the kinetic energy and compare the collected energy of the NVSS.